

Evaluating the effectiveness of Advanced Programme Mathematics in preparing learners for university Mathematics

Hester du Plessis

Thesis presented in partial fulfilment of the requirements

for the degree of Master of Education

in the Department of Curriculum Studies

Faculty of Education



at

Stellenbosch University

Supervisor: Dr Mdutshekelwa Ndlovu

Co-supervisor: Prof Magda Fourie- Malherbe

March 2015

DECLARATION

I, the undersigned, hereby declare that the work contained in this thesis is my own original work and that I have not previously in its entirety or in part submitted it at any university for a degree.

Signature:

Date:

25/02/2015

ABSTRACT

In today's hi-tech global economy the fields of science, technology and engineering are becoming increasingly and undeniably central to economic growth and competitiveness, and will provide many future jobs. Qualifications in Mathematics are crucial gateways to further education and will provide access to the Science, Technology, Engineering and Mathematics (STEM) industries.

This study focuses on the optional course in Mathematics, called Advanced Programme Mathematics (APM), which is offered and assessed by the Independent Examination Board in the final three years of high school in South Africa.

At present, the South African school system does not adequately prepare students for the transition from school to university Mathematics, and APM has been designed to address this gap. The research question set by this study is: To what extent does the APM course succeed in preparing learners for the rigour of first-year Mathematics in the STEM university programmes?

The sample group of 439 students was selected from the 2013 cohort of first-year Mathematics students at Stellenbosch University. First, an analysis of the relevant curricula was undertaken, and then an empirical investigation was done to determine the differences in performance between first and second semester examinations of first-year university Mathematics students who took APM, and those who did not. This was followed by an investigation by means of a questionnaire into the perceptions of students on how effective APM was in easing the transition from school to university Mathematics. The research was designed according to the Framework for an Integrated Methodology (FraIM) of Plowright (2011).

From an extensive international literature study, it appears that APM is definitely a predictor of post-secondary success. Since no formal research has been recorded to support this claim, this study aims to provide a sound answer to whether APM is advantageous. The effect size results of this study show that APM marks of students explain 68% of the achievement in first-semester university Mathematics when combined with NSC Mathematics marks in a general regression model.

There is a significant difference between the marks of students who took APM and those who did not in first-semester university Mathematics, specifically across the National Senior Certificate (NSC) Mathematics mark categories of 80-100%.

APM course-taking leads to confidence in Mathematics, which combined with good domain knowledge of calculus, ease the transition from school to university Mathematics.

The study recommends that not only students who intend pursuing a career in the STEM industries should take the APM course, but also those who intend to apply for admission to any other tertiary studies, as the cognitive and other skills provided by APM will give them the required edge to perform well in higher education. Schools are called upon to provide access to APM for mathematically gifted students, and teachers and guidance counsellors should encourage learners to enrol for AMP. This will enable them to share in the manifold academic and personal benefits accruing from the course, and to help alleviate the critical shortage of graduates in careers requiring a strong Mathematics background in South Africa.

OPSOMMING

In die hoë-tegnologie-wêreld ekonomie van vandag word die gebiede van wetenskap, tegnologie en ingenieurswese toenemend en onmiskenbaar die kern van ekonomiese groei en mededingendheid wat in die toekoms baie werkgeleenthede sal bied. Kwalifikasies in Wiskunde open beslis baie deure na verdere opleiding en verleen toegang tot die Wetenskap-, Tegnologie-Ingenieurswese- en Wiskunde-industrieë.

Hierdie studie fokus op die opsionele kursus in Wiskunde, genaamd Gevorderde Program Wiskunde (GPW), wat deur die Onafhanklike Eksamenraad aangebied en geassesseer word in die laaste drie jaar van hoërskoolonderrig in Suid-Afrika.

Tans berei die Suid-Afrikaanse skoolstelsel nie studente genoegsaam voor vir die oorgang van skool- na universiteitswiskunde nie en GPW is ontwerp om hierdie gaping te oorbrug. Die navorsingsvraag wat hierdie studie stel, is: In watter mate slaag die GPW-kursus daarin om leerders voor te berei vir die streng vereistes van eerstejaar-Wiskunde in die Wetenskap-, Tegnologie- Ingenieurswese- en Wiskunde-universiteitsprogramme?

Die toetsgroep van 436 studente is gekies uit die 2013-groep eerstejaar-Wiskundestudente aan Stellenbosch Universiteit. Aanvanklik is 'n analise van die relevante leerplanne onderneem, waarna 'n empiriese ondersoek gedoen is om die verskille in prestasie in die eerste en tweede semester eksamens vas te stel tussen eerstejaar-Wiskundestudente op universiteit wat wel GPW geneem het en diegene wat dit nie geneem het nie. Dit is gevolg deur 'n ondersoek deur middel van 'n vraelys na die persepsies van studente oor hoe effektief GPW was om die oorgang van skool- na universiteitswiskunde te vergemaklik. Die navorsing is ontwerp op grond van 'n model vir 'n geïntegreerde metodologie van Plowright (2011).

Dit blyk uit 'n uitgebreide studie van internasionale literatuur dat GPW definitief 'n voorspeller van post-sekundêre sukses is. Aangesien geen formele navorsing om hierdie aanspraak te ondersteun nog op skrif gestel is nie, poog hierdie studie om 'n deurdagte antwoord te verskaf op die vraag of GPW wel tot voordeel van studente is.

Die effek grootte resultate van hierdie studie dui aan dat die GPW-punte van studente 68% van prestasie in Wiskunde in die eerste semester op universiteit verduidelik as dit in 'n algemene regressiemodel met die Nasionale Senior Sertifikaat (NSS) punte gekombineer word. Daar is 'n beduidende verskil tussen die Wiskundepunte van studente wat GPW geneem het en diegene wat dit nie geneem het nie in die eerste semester op universiteit, veral in die NSS-Wiskundepuntekategorieë van 80-100%.

Om die GPW-kursus te neem, lei tot selfvertroue in Wiskunde, wat saam met 'n goeie kennis van die Differensiaalrekening-domein, die oorgang van Wiskunde vanaf skoolvlak na universiteitsvlak vergemaklik.

Op grond van die studie beveel die navorser aan dat nie slegs studente wat 'n loopbaan in Wetenskap-, Tegnologie- Ingenieurswese- en Wiskunde-rigtings wil volg, die GPW-kursus behoort te volg nie, maar ook diegene wat vir toelating tot enige ander tersiêre studie wil aansoek doen, aangesien die kognitiewe en ander vaardighede wat GPW ontwikkel, hulle die nodige voorsprong sal bied om goed te vaar in verdere studie. Skole word aangemoedig om toegang tot GPW aan wiskundig begaafde leerlinge te verskaf en onderwysers en loopbaanraadgewers behoort leerlinge aan te moedig om vir GPW in te skryf. Sodoende kan hulle deel in die vele akademiese en persoonlike voordele wat die kursus bied, en help om die kritieke tekort aan gegradueerdes in die studierigtings waar 'n sterk Wiskunde agtergrond 'n vereiste is, te help verlig.

ACKNOWLEDGEMENTS

The writing of this thesis was not only a challenge for me, but also affected the lives of a number of people around me. I would like to express my gratitude to these people:

My supervisors Dr Mdutshekelwa Ndlovu and Prof Magda Fourie-Malherbe, for the role they played as mentors in sharing their wealth of knowledge and experience in the higher education field with me, while also providing critical analysis and encouragement.

My father Martin, brother Bernard, and friend Michael for their patience and invaluable assistance from the initial structuring of my thoughts to the final product.

Annie, Marnus, and my mother, Ellen, for caring, encouraging and believing in me!

My close friends and colleagues for their interest and encouragement.

Prof Daan Nel for much appreciated help with the statistical analyses and Marinda van Rooyen for her help with the distribution of the questionnaire.

My friend Loumarie Kistner for unselfishly sharing her expertise in statistics and technical editing on numerous occasions.

The Mathematics Department of Stellenbosch University, the Independent Examination Board, as well as Jean de Villiers for their willingness to provide data and information needed.

In addition, I would like to thank Stellenbosch University for providing the institutional support within which this research could be undertaken.

DEDICATION

I dedicate this thesis to all the Advanced Programme Mathematics learners whom I have taught over the past six years. Your enthusiasm and energy made teaching and life worthwhile!

ACRONYMS

ANA	Annual National Assessments
AP	Advanced Placement
APM	Advanced Programme Mathematics
CASS	Continuous Assessment
CDE	Centre for Development and Enterprise
CIE	Cambridge International Examinations
CHE	Council on Higher Education
DBE	Department of Basic Education
DHET	Department of Higher Education and Training
DoE	Department of Education
FET	Further Education and Training
FYGPA	First year grade point average
GCE	General Certificate of Education
GET	General Education and Training
HE	Higher Education
HESA	Higher Education South Africa
HG	Higher grade
IBDP	International Baccalaureate Diploma Programme
IEB	Independent Examination Board
NCS	National Curriculum Statement
NBT	National Benchmark Tests
NQF	National Qualifications Framework
NSC	National Senior Certificate
NRC	National Research Council
OBE	Outcomes-based Education

PIRLS	Progress in International Reading and Literacy Studies
SACMEQ	Southern and Eastern African Consortium for Monitoring Educational Quality
SAQA	South African Qualifications Authority
SC	Senior Certificate
SES	Socio-economic status
SET	Science, Engineering and Technology
SG	Standard grade
SAIRR	South African Institute for Race Relationships
STEM	Science, Technology, Engineering and Mathematics
SU	Stellenbosch University
TIMSS	Trends in International Mathematics and Science Study
UCT	University of Cape Town
UNESCO	United Nations Educational, Scientific, and Cultural Organisation
USA	United States of America

TABLE OF CONTENTS

DECLARATION	ii
ABSTRACT	iii
ACKNOWLEDGEMENTS	vii
DEDICATION	viii
ACRONYMS	ix
TABLE OF CONTENTS	xi
LIST OF FIGURES	xvii
LIST OF TABLES	xix
CHAPTER 1 INTRODUCTION AND ORIENTATION TO THE STUDY	1
1.1 INTRODUCTION	1
1.2 CONTEXT IN WHICH THE RESEARCH TOOK PLACE	2
1.2.1 <i>National context: Education in South Africa</i>	2
1.2.2 <i>Policy context</i>	11
1.2.3 <i>Organisational context: Stellenbosch University</i>	12
1.2.4 <i>Professional context: The researcher's experience</i>	13
1.3 PROBLEM STATEMENT	14
1.4 PURPOSE AND SIGNIFICANCE OF THE STUDY	16
1.5 RESEARCH QUESTION	17
1.6 OBJECTIVES	17
1.7 THE RESEARCH PROCESS	18
1.7.1 <i>Research design and methodology</i>	18
1.7.2 <i>The participants and research data</i>	19
1.7.3 <i>Analysis of the data</i>	19
1.8 BRIEF CHAPTER OVERVIEW	20

CHAPTER 2	LITERATURE REVIEW	21
2.1	INTRODUCTION	21
2.2	THE CONCEPT OF AN ADVANCED PLACEMENT COURSE	22
2.2.1	<i>AP courses in the USA.....</i>	<i>23</i>
2.2.2	<i>Other international courses with curricula similar to those of AP courses.....</i>	<i>24</i>
2.2.3	<i>AP courses in South Africa.....</i>	<i>26</i>
2.3	THE HISTORY OF AP PROGRAMMES IN THE USA.....	27
2.4	THE ADVANCED PROGRAMME MATHEMATICS COURSE IN SOUTH AFRICA.....	30
2.5	THE HISTORY OF APM IN SOUTH AFRICA.....	31
2.5.1	<i>Curriculum development in South Africa in the post-apartheid era..</i>	<i>31</i>
2.5.2	<i>The development of the APM course</i>	<i>32</i>
2.6	GLOBAL CONCERNS ABOUT AP PROGRAMMES CONCEPTUALISED IN SOUTH AFRICAN CONTEXT.....	34
2.6.1	<i>Curriculum issues.....</i>	<i>35</i>
2.6.2	<i>Issues related to increased access</i>	<i>36</i>
2.6.3	<i>The financial cost aspect of AP programmes</i>	<i>38</i>
2.6.4	<i>AP courses as an indicator of a school's quality</i>	<i>39</i>
2.6.5	<i>Non-academic implications of enrolling for an AP course</i>	<i>40</i>
2.6.6	<i>The possible academic benefits of AP</i>	<i>41</i>
2.7	SUMMARY	46
CHAPTER 3	THEORETICAL FRAMEWORK.....	47
3.1	INTRODUCTION	47
3.2	CHOOSING A RESEARCH FRAMEWORK	47
3.2.1	<i>Phenomena in Mathematics education</i>	<i>47</i>
3.2.2	<i>The nature of research frameworks.....</i>	<i>49</i>
3.2.3	<i>Theories on mathematical learning</i>	<i>51</i>

3.3	PERSPECTIVES ON CURRICULA	56
3.3.1	<i>The curriculum and student learning</i>	56
3.3.2	<i>The South African OBE Mathematics curriculum</i>	57
3.3.3	<i>Thinking skills required in the transition from school to university Mathematics</i>	61
3.4	STUDENT SUCCESS AT UNIVERSITY	64
3.4.1	<i>The concept 'student success'</i>	64
3.4.2	<i>Prominent theories on student success</i>	65
3.4.3	<i>Variables predicting students' academic success</i>	67
3.4.4	<i>Cognitive ability as a determinant of academic achievement</i>	67
3.4.5	<i>Non-cognitive predictors of academic achievement</i>	68
3.5	THEORIES ON STUDENT SUCCESS IN MATHEMATICS	70
3.5.1	<i>Bandura's self-efficacy theory</i>	71
3.5.2	<i>The self-confidence theory of Stankov</i>	77
3.5.3	<i>The mathematical empowerment theory of Ernest</i>	79
3.5.4	<i>Ackerman, Kanfer and Beier's findings on APM as domain knowledge</i>	82
3.5.5	<i>Synthesis</i>	83
3.6	A CONCEPTUAL FRAMEWORK FOR THIS STUDY	84
3.7	SUMMARY	85
CHAPTER 4	RESEARCH METHODOLOGY	87
4.1	INTRODUCTION	87
4.2	PLANNING THE RESEARCH	87
4.2.1	<i>Traditional research approaches and paradigms</i>	88
4.2.2	<i>The research design for this study</i>	91
4.2.3	<i>The research question</i>	94
4.3	CURRICULAR ANALYSES	94
4.3.1	<i>The concept</i>	94

4.3.2	<i>The sample documents</i>	95
4.3.3	<i>Evaluating the evidence</i>	96
4.3.4	<i>Ethical issues</i>	97
4.4	COMPARISON OF STUDENT PERFORMANCE	97
4.4.1	<i>Introduction</i>	97
4.4.1	<i>Participants and sampling</i>	98
4.4.2	<i>Research instruments</i>	107
4.4.3	<i>Instrument reliability and validity</i>	107
4.4.4	<i>Data collection procedures</i>	109
4.4.5	<i>Data analysis procedures</i>	109
4.5	SURVEY QUESTIONNAIRE	112
4.5.1	<i>Participants and sampling</i>	112
4.5.2	<i>Research Instruments</i>	113
4.5.3	<i>Method of data collection</i>	115
4.5.4	<i>Analysis of data</i>	115
4.6	ETHICAL CONSIDERATIONS	117
4.6.1	<i>Ethical issues associated with the research question</i>	117
4.6.2	<i>Ethical issues associated with the content</i>	118
4.6.3	<i>Ethical issues associated with the participants</i>	118
4.6.4	<i>Ethical issues regarding data and data analysis and reporting</i>	118
4.7	LIMITATIONS	119
4.8	CONCLUSION	119
CHAPTER 5 PRESENTATION AND INTERPRETATION OF RESEARCH RESULTS		120
5.1	INTRODUCTION	120
5.2	SECONDARY RESEARCH QUESTION A	121
5.2.1	<i>Overview of each curriculum</i>	121

5.2.2	<i>Comparison of the nature of the curricula of NSC Mathematics and APM</i>	124
5.2.3	<i>Comparison of content of syllabi</i>	125
5.2.4	<i>Comparison of the content weighting of Calculus in different curricula</i>	130
5.2.5	<i>A comparison of examination mark totals and time allocation</i>	131
5.2.6	<i>Cognitive demands.....</i>	132
5.2.8	<i>Summary of Section 5.2</i>	141
5.3	SECONDARY RESEARCH QUESTION B	142
5.3.1	<i>Descriptive statistics of the two groups in Sample A when entering university</i>	143
5.3.2	<i>Descriptive statistics of APM Yes group and APM No group in Sample A during the first year of university</i>	152
5.3.3	<i>Inferential statistics of Sample A</i>	162
5.3.4	<i>Summary of Section 5.3.....</i>	167
5.4	SECONDARY RESEARCH QUESTION C	167
5.4.1	<i>Profile of students in Sample B</i>	168
5.4.2	<i>Perception of the students in Sample B of the transition from school Mathematics to university Mathematics.....</i>	171
5.4.3	<i>Students' perceptions of the effect of APM course-taking</i>	171
5.4.4	<i>Summary of Section 5.4.....</i>	175
5.5	VOICES OF THE STUDENTS IN THE OPEN-ENDED QUESTION.....	176
5.5.1	<i>Comprehension.....</i>	177
5.5.2	<i>Enjoyment of Mathematics</i>	177
5.5.3	<i>Work tempo</i>	178
5.5.4	<i>Skills.....</i>	178
5.5.5	<i>Mathematics anxiety.....</i>	179
5.5.6	<i>Confidence</i>	179
5.5.7	<i>Summary of Section 5.5.....</i>	179

5.6 SUMMARY AND INTEGRATION OF RESEARCH RESULTS	180
CHAPTER 6 CONCLUSIONS.....	183
6.1 INTRODUCTION AND SUMMARY	183
6.2 FINDINGS AND CONCLUSIONS.....	183
6.3 LIMITATIONS OF THIS STUDY	186
6.4 RECOMMENDATIONS	187
6.5 A FINAL WORD.....	189
REFERENCES	190
APPENDIX A: LETTER OF ETHICAL CLEARANCE.....	232
APPENDIX B: INSTITUTIONAL PERMISSION.....	233
APPENDIX C: PERMISSION FROM IEB	234
APPENDIX D: QUESTIONNAIRE.....	235
APPENDIX E: EXAMINATION PAPERS	238
ADVANCED PROGRAMME MATHEMATICS (2012)	238
MATHEMATICS 114 (2013).....	258

LIST OF FIGURES

Figure 3.1	Conceptual framework.....	85
Figure 4.1:	The FraIM.....	93
Figure 4.2:	Demographic variables of the target population	100
Figure 4.3:	Frequency tables of data on cognitive variables of target population	102
Figure 4.4:	Biographical data of sample compared to that of target population	105
Figure 5.1:	Gender distribution between APM Yes and APM No groups.....	145
Figure 5.2:	Grade 12 year of APM Yes and APM No groups	145
Figure 5.3:	Ethnicity of the APM Yes and APM No groups.....	146
Figure 5.4:	Type of high school attended	147
Figure 5.5:	Grade 12 average marks across different performance categories as a percentage of Sample A	149
Figure 5.6:	NSC Mathematics marks across different performance categories as percentage of Sample A	150
Figure 5.7:	Mathematics Paper 3 marks across different performance categories as percentage of Sample A	151
Figure 5.8:	NBT Mathematics marks across the different performance categories as a percentage of Sample A.....	151
Figure 5.9:	First-semester Mathematics marks across different performance categories as percentage of Sample A.....	154
Figure 5.10:	Second-semester Mathematics marks across different performance categories as percentage of Sample A.....	154
Figure 5.11:	Difference between groups in first-semester Mathematics across NSC Mathematics categories	156

Figure 5.12: Difference between groups in second-semester Mathematics performance across NSC Mathematics categories	158
Figure 5.13: Difference between two groups in first-semester Mathematics performance across NBT Mathematics categories	160
Figure 5.14: Difference between two groups in second-semester Mathematics performance across NBT Mathematics categories	161
Figure 5.15: Number of students repeating a Mathematics course	168
Figure 5.16: Level of difficulty experienced in the transition from school to university Mathematics	169
Figure 5.17: Distribution of the NSC Mathematics marks for Sample B	170
Figure 5.18 Type of school attended	170
Figure 5.19: The relationship between confidence and first-semester Mathematics marks	174
Figure 5.20: The relationship between confidence and second-semester Mathematics marks	175

LIST OF TABLES

Table 2.1:	Comparison of challenges facing higher education in USA and South Africa	34
Table 4.1:	Sample size and target population size	104
Table 4.2:	Results of chi-square test to determine gender representation	106
Table 4.3:	Results of chi-square test to determine ethnic representation	106
Table 4.4:	Results of chi-square test to determine matric year representation....	106
Table 4.5:	Results of chi-square test to determine school type representation	106
Table 4.6:	Summary of the appropriate statistical test for each research objective	110
Table 4.7:	Constructs in the questionnaire	114
Table 4.8:	Methods used to analyse the research objectives underlying the questionnaire	116
Table 5.1:	Comparison of NSC Mathematics and Advanced Programme Mathematics	124
Table 5.2:	Comparison of the content of NSC Mathematics, APM and first-year university Mathematics	125
Table 5.3:	Examination mark totals and time allocation	131
Table 5.4:	Learning outcomes of NSC Mathematics APM, Mathematics 114 and Mathematics 115 curricula.....	133
Table 5.5:	Learning outcomes of NSC Mathematics and APM curricula according to the cognitive levels in revised Bloom's taxonomy. ...	135
Table 5.6:	Taxonomical differentiation of Grade 12 question	138

Table 5.7:	Summary of verbs used to describe skills in different examination papers	139
Table 5.8:	Demographic profile of Sample Group A	144
Table 5.9:	The mean and confidence intervals for the previous learning variables	148
Table 5.10:	The mean and confidence intervals for first-year Mathematics	153
Table 5.11:	Anova and Bonferroni tests for difference in first-semester Mathematics performance across NSC Mathematics categories .	155
Table 5.12:	Anova and Bonferroni tests for difference in second-semester Mathematics performance across NSC Mathematics categories .	157
Table 5.13:	Anova and Bonferroni tests for difference in first-semester Mathematics across NBT Mathematics categories.....	159
Table 5.14:	Anova and Bonferroni tests for difference in second-semester Mathematics across NBT Mathematics categories.....	161
Table 5.15:	Results of correlation analysis.....	162
Table 5.16:	General regression model on first semester performance.....	164
Table 5.17:	Test of whole model on first semester performance	164
Table 5.18:	General regression model on second-semester performance.....	166
Table 5.19:	Test of whole model on second-semester performance	166
Table 5.20:	General regression model on second-semester performance (with first-semester)	166

CHAPTER 1

INTRODUCTION AND ORIENTATION TO THE STUDY

“We are to prepare independent thinkers who will forge new paths with skills they can transfer into contexts we haven’t even imagined yet”

(Croucamp, 2013)

1.1 INTRODUCTION

South Africa has many young people with exceptional talents in Mathematics. These young people hold extraordinary potential for enriching our society by contributing creative products and competing in global economies. In their later life as adults, they will hold important leadership roles and be entrusted with obligations and resources for making critical decisions about individual and organisational well-being. They constitute the far edge of a population whose continued success will be globally acknowledged in the near future (Kell, Lubinski & Benbow, 2013).

In today’s hi-tech global economy, the Science, Technology, Engineering and Mathematics (STEM) industries are becoming increasingly central to economic competitiveness and growth, and will provide many of the future jobs. Knowledge of Mathematics and qualifications in Mathematics are increasingly important gateways to further and higher education (HE).

This study presents an evaluation of a programme in Mathematics, called Advanced Programme Mathematics (APM). It is offered in the final three years of high school as an option for South African mathematically gifted learners with enthusiasm for Mathematics. Advanced Programme Mathematics is aimed at increasing the number of learners who, through competence and desire, enter higher education to pursue careers in the STEM fields (DoE, 2006).

This chapter describes the context in which the research question was formulated, provides an overview of the designing, planning and carrying out of the research and finally positions the research questions within the scope of the research.

1.2 CONTEXT IN WHICH THE RESEARCH TOOK PLACE

There are five different contexts contingent to carrying out any research, namely the theoretical, national, policy, organisational and professional ones (Plowright, 2011). The next section will discuss the first four contexts and Chapter 3 will sketch the theoretical context.

1.2.1 *National context: Education in South Africa*

(a) *The current situation*

The participants in this study entered university in 2013. In the same year the Centre for Development and Enterprise (CDE) published a report, based on independent research by university-based experts Spaull and Simkins, which painted a dark picture of the quality of education in South Africa. The current system is failing the majority of South African youth. With the exception of a wealthy minority, most South African learners are not where they should be in terms of the curriculum and have not reached the appropriate normal milestones of numeracy and literacy. “The education system is grossly inefficient, severely underperforming and egregiously unfair” (Spaull, 2013, p. 3).

South Africa has a population of roughly 50 million, of which 80% is designated African, 9% coloured, 9% white and 3% South African Indian (Statistics South Africa, 2013). By 2013 race was no longer the only indicator of social privilege. Recently there has been a significant sector of the middle class and very rich, that is African, coloured or Indian. This is reflected in shifts in terms of behaviour and affiliation amongst students in HE, where a tendency to see oneself in terms of social class rather than only in terms of race is increasing (Leibowitz & Bozalek, 2014).

Poverty is still one of the largest socio-economic challenges facing the country (Marais, 2011). The Gini co-efficient for South Africa, measuring the distance between the

richest and poorest individuals in a country, is one of the highest in the world (Bhorat, Tseng, Stanwix, 2014). Not only society, but also schools, are very unequal in South Africa, with effectively two types of schools: functional schools (25%) and dysfunctional schools (75%). These schools are still mostly differentiated on the base of race (Van der Berg, 2008), but also differ in terms of accountability, competence of school management, culture of learning, teachers' knowledge of content, teacher absenteeism, coverage of curriculum and homework, dropout rate and performance on national tests (Spaull, 2013).

From a global perspective of the situation, South Africa's education is not very inspiring either. For some years already, South Africa has lagged behind in three of the comparative international studies. The three main international tests of educational achievement in which South Africa participates are the Trends in International Mathematics and Science Study (TIMSS), Progress in International Reading and Literacy Studies (PIRLS), and Southern and Eastern African Consortium for Monitoring Educational Quality (SACMEQ). These tests show how the performance of South African learners has changed over time relative to earlier groups of South African learners, and relative to other countries participating in these studies.

In the most recent (2007) SACMEQ (Grade 6 numeracy and literacy) tests, South African learners ranked 8th out of 14 countries on the African continent for Mathematics, and many South African Mathematics teachers had below basic levels of content knowledge (Spaull, 2013). In the 2006 PIRLS, South African learners achieved the lowest score of 45 countries that participated, including other middle-income countries such as Morocco (Spaull, 2013). In terms of the quality of Mathematics and Science education, the World Economic Forum's 2013-2014 Report ranks South Africa second-last out of 144 countries, above only Yemen (Schwab, 2013).

Locally there are three sets of Annual National Assessments (ANA's) of literacy and numeracy, externally evaluated and nationally standardised, which point to alarmingly low levels of achievement in the fundamental building blocks of learning, despite the Department of Basic Education's (DBE) pointing to some progress, especially in the lower grades. One of the most recent dramatic results was that only 2.3% of Grade 9

learners in 2012 achieved more than 50% in Mathematics, with an average mark of 13% (Yeld, 2012).

Whether the results of the school-leaving (NSC) examination can be seen as an accurate indication of the quality of education in South Africa is a controversial issue. It is widely criticised on aspects such as the pass rate, which does not take into account drop-out before Grade 12, nor the fact that more learners are taking easier subjects such as Mathematical Literacy instead of the more difficult Mathematics (Spaull, 2013).

The 2012 NSC results indicate an overall 73.9% pass rate, but only 26.6% of those tested qualified for bachelor degree studies (DBE, 2013). The high levels of inequality between functional and dysfunctional schools are also reflected in these results (Spaull, 2013). Morrow (2009) reflected on the social, economic and cultural influences on schooling in contemporary South Africa, and noted that race, ethnicity, language and class cohere as factors at school level to hinder students from educationally disadvantaged backgrounds from gaining epistemological access once they reach university. The notion 'epistemological access' will be discussed in a further section of this study.

It appears that attempts at reform and increased expenditure on schooling do not correlate with equally improved outcomes. The 2012 assessment report of the DBE reports a list of interventions already in place to address the needs of education in South Africa. Various analysts have described the problems and the reasons for them, which are complex, deep and extensive (Bray, Gooskens, Kahn, Moses & Seekings, 2010; Lam, Ardington & Leibbrandt, 2013). Two of the major underlying problems, are the teacher's lack of knowledge about what they teach and the fact that many learners are learning through the medium of first additional language (Yeld, 2012). "Fixing education is a complex, long and challenging undertaking" (Yeld, 2012, para. 23).

Thirty percent of the South Africa population is involved in one of the five major components of the educational system. These areas are early education, basic education, special education, Further Education and Training (FET) colleges and universities (Simkins, 2013). This study will focus on the transition from basic education to HE.

(b) Mathematics education in South Africa

It is to be expected that the problematic educational situation in South Africa reflects on the performance of learners in Mathematics. “The teaching of Mathematics in South African schools is among the worst in the world” (Bernstein, 2013, p. 3). The TIMSS (2011) report showed that South African learners have the lowest performance among all 21 middle-income countries that participated and that they perform worse than many low-income African countries. In a recent CDE report, Bernstein (2013, p. 3) summarises the urgency of the situation:

efforts fail to address the wider deficiencies in Mathematics education. Vast improvements in this area of the public schooling system are vital to South Africa’s future socio-economic prospects: for the learners as well as the development of the country as a whole.

Many indicators on school performance and teaching reveal largely unacknowledged poor teaching of Mathematics in the majority of South African schools. South African Mathematics teachers’ competencies compare poorly to those of their counterparts in other Eastern and Southern African countries – they mostly rank at the bottom end of the list. As a result, learner results will not quickly be remedied (Spaull, 2013). The CDE report claims a rapid increase in private extra Mathematics classes, partly in response to the poor teaching in public schools (Bernstein, 2013).

Two other major problems are teacher complacency and a lack of accountability in various areas. Teacher complacency is linked to the way in which many teachers are appointed – often not on merit. There is little accountability to parents in the majority of the school system, little accountability between teachers and the Department of Education, and teacher unions abuse power and act unprofessionally. Furthermore, South Africa’s extremely high unemployment is closely linked to the quality of schooling and numeracy and mathematics competency in particular (Bernstein, 2013).

The CDE warns that improving mathematics teaching and learning in public schools will not happen fast, but must be seen as a matter of urgency. The South African government has begun to accept that there is a crisis requiring immediate intervention,

yet its extent and depth is often underestimated in favour of reports of 'progress' (Bernstein, 2013).

The poor results in the final school leaving examination in Mathematics, the National Senior Certificate (NSC) Mathematics, affect learners' ability to obtain the required grades for university admission. The number of Mathematics passes at 40% and above (many of which are inadequate for HE success) remains well below the current intake needs (Fisher & Scott, 2011). Half of the Grade 12 learners in 2012 who wrote Mathematics scored lower than 30%, and one out of three less than 20%. One fifth of students achieved more than 50%, and only 6%, mostly from former white schools, achieved more than 69%, which is the minimum requirement to enrol for any university Mathematics course (DBE, 2013). Not every student needs to obtain a university degree, but the pool of competent candidates that gain access is too small. This situation impedes successful growth in many programmes that are essential for the economic growth of the country.

The correlation between the performance in school-level Mathematics and the outcomes in introductory quantitatively oriented university courses has been widely researched internationally as well as in South Africa (Nel & Kistner, 2009; Rankin, Schöer; Sebastiao & Walbeek, 2012; Smith & Naylor, 2001). Research shows that South African universities are unsure about the usefulness of NSC Mathematics marks as predictors of academic performance at university level (Schöer, Ntuli, Rankin, Sebastiao & Hunt, 2010). Prior to 2008 the Senior Certificate (SC) had a good track record as a relatively robust predictor of ability (Scott, Yeld & Hendry, 2007). With the introduction of the National Senior Certificate (NSC), concerns were raised after the 2008 NSC Mathematics results showed that an increased number of students across the country achieved more than 80% in the final Mathematics examination – a situation repeated in 2009.

Instead of differentiating students according to their abilities, the new NSC Mathematics marks compress students with a wide range of abilities and disabilities into a very narrow range of percentage marks. This could be because there is no longer any

differentiation between higher, standard and lower grades in the NSC syllabus, as was the case in the SC syllabus (Schöer et al., 2010).

The 2008 NSC Mathematics results confirmed the notion of grade inflation, especially in the lower performance group (Nel & Kistner, 2009). This means that the NSC marks obtained by students do not match their actual performance. Grade inflation is normally associated with falling standards, but it can also be explained by a number of factors, such as a change in curriculum, improvements in the manner of examining, increased use of continuous assessment or the distribution of sample papers (Govender & Moodley, 2012). The consequence of grade inflation is that it leads to lack of trust in the predicting value of the NSC. Students leave school with the expectation that their school-leaving marks signal their true ability and they gain access to courses in the STEM field, but are in fact underprepared and do not have the ability to be successful in these courses.

Wolmarans, Smit, Collier-Reed and Leuther (2010) argue that the poor performance of first-year students since the introduction of the NSC is not a sudden or dramatic shift, but rather part of a gradual deterioration in the preparedness of these students for the demands of HE study. Nel, Kistner and Van der Merwe (2013), in a preliminary report on the NSC 2014 intake, raise concerns about the weak algebraic competencies of learners, the low literacy levels (learners cannot read the questions), the absence of higher-order problem-solving skills and the learners' inability to work without a calculator.

The discussion on the NSC Mathematics marks should, however, not distract attention from the main issue of Mathematics teaching and learning at school level and the assumptions made by HE about the mathematical skills of incoming students. The universities should also not merely shift the blame, but take ownership of the issue.

In mid-2004, institutional representatives in HE laid the foundation for the National Benchmark Tests Project. This project piloted in February 2009 at selected HE institutions. The National Benchmark Tests (NBT) provides meaningful information about students' educational needs on their entry to HE. Written before or on entry to an institution, they are used in some way by the majority of South Africa's universities (Yeld, 2009). These NBT scores can play a dominant role in ensuring that students

admitted to first-year programmes have at least a reasonable chance of success when channelled into their appropriate curricular routes (Du Plessis & Gerber, 2012). In October 2012 representatives of each of the country's 23 public universities determined the *new* NBT tests that would take effect for the 2014 cohort of first-year students.

The NBT Mathematics test is explicitly designed to provide a snapshot of the mathematical competencies of test writers at some point in time. The knowledge and skills assessed relate to school mathematical content that is relevant to HE, and the tests attempt to determine how well relevant mathematical concepts have been understood and can be applied (Griesel, 2006).

On the one side of this gap between school and university Mathematics, the HE institutions might raise their entrance requirements dramatically and on the other side, educational authorities might set much more difficult examination papers. In the process, the school-learner is the party who is affected negatively. It is into this gap that APM fits as a better alternative to these two options, with the focus on the learner and his transition between school and HE. (In this study references to one gender are inclusive of the other gender.)

(c) Transition from basic education to higher education

Although there is a strong thrust to provide access to HE to students who were previously excluded because of apartheid, the pool of students able to gain access is still exceedingly small. Current statistics have it that out of every 100 learners who start school in grade one, only 50 will make it to grade 12, 40 will pass and only 12 will qualify for university (Spaull, 2013). Prior education is a key factor influencing how students learn and continue to learn.

Enrolments at universities are growing fast and the consequent challenges, such as student retention and maintaining high standards, are increasing. There has been a degree of movement towards transformation in terms of access to HE, but less in terms of success (Crouch & Vinjevold, 2006; Leibowitz & Bozalek, 2014; Simkins, 2013).

The current state of access, success and throughput in HE is revealed in a recent report of the Council on Higher Education (CHE) on curriculum reform in South Africa. This

report (Scott, Ndebele, Badsha, Figaji, Gevers & Pityana, 2013) argues the need for radical changes to the undergraduate curriculum. The following observations in the report are of interest for this study:

The HE participation rate as a percentage of the group from 20–24 years old has increased from 15% to 18% in 2010 and there has been significant progress in improving African and coloured participation in HE, although it is still low. The intake of African and coloured students represents the top 10% of the African and coloured youth, which means they must have high potential to succeed. First-year attrition is very high. In 2006 nearly 42 000 students of the first-time entering total of 127 000 dropped out, which is one out of every three.

In the best performing cohort to date, the 2006 cohort, only 35% of the intake graduated in five years, and it is estimated that 55% will never graduate. Graduation rates for the qualifications BEng and BSc were both 23%, and the white graduation rate in engineering in the 5- year course was 2.4 times (139%) higher than the African one. The overall effect of the performance patterns is that only 5% of African and coloured youth are succeeding in HE. This represents an “unacceptable failure to develop talent in groups where realisation of potential is most important” (Scott et al., 2013, p. 51). South Africa is not producing sufficient graduates to meet national needs in terms of economic and social development and that the country’s intellectual talent is not being developed sufficiently (Scott et al., 2013).

The CHE report concludes that poor academic preparation at school is the main reason for poor university performance and it expresses doubt about whether the dysfunctional school system will be able to produce the numbers of adequately prepared school-leavers that HE requires in the near future. In the light of these circumstances, the report introduces a new curriculum structure. The authors propose that all current three-year degrees and diplomas, as well as current four-year professional bachelor's degrees, should be increased by one year, while allowing students who can complete a programme in less than the formal time to do so (Scott et al., 2013). Critics claim that this report’s analysis is unsound and narrow and that HE must take part of the blame for the under-preparedness of students entering universities. They name lack of adequate

student services, unqualified academic staff and proper academic support and mentoring as factors contributing to the high failure rates (Selamolela & Masondo, 2013).

Many South African universities have developed access programmes as alternative routes to university admission. Calls for foundation courses to bridge knowledge gaps, curriculum extension to allow some students more time to complete their studies and co-curricular and academic literacy programmes to develop competencies and skills continue to be made (Hlalele, 2010). Despite all these efforts, participation to ensure access is still inadequate (Leibowitz & Bozalek, 2014).

Yeld (2010) suggests that South Africa has to continue emphasising access as a national goal and needs to intensify its efforts in this regard. This must however always be access for success rather than access for participation.

(d) Education for the gifted in South Africa

“Educational provision for gifted learners should reflect our understanding of what it means to be gifted” (Eyre & Lowe, 2012, p. 1). ‘Giftedness’ is a questionable construct that has posed a challenge to many scholars over the years. Definitions of ‘giftedness’ are linked with synonyms such as ‘high ability’, ‘aptitude’ or ‘talent’, and they often carry a long history of cultural use, are tainted with emotionalism and even associated with elitism (Monks & Katzko, 2005).

For the sake of this study, gifted learners will be described as those learners, who in general, learn quickly, exhibit great efficiency in problem-solving, understand advanced and complex concepts in a variety of reasoning domains and are proficient and creative producers of thoughts (Renzulli, 1983). These learners need access to broad, balanced and challenging curriculum opportunities that make provision for critical thinking, creative thinking, increased independence, problem solving, ability reflection and self-knowledge (Eyre & Lowe, 2012). Therefore, the interaction between gifted students and their environment produces situations in the classroom requiring curricular and

instructional differentiation to enable them to fulfil their potential (Foust, Hertberg- Davis & Callahan 2009).

Gifted education has often been criticised for violating the principles of equity (Monks & Katzko, 2005). Persson (2014) asks the very relevant question, given the South African history: “Does gifted education affect societal inequality, and does societal inequality suppress and/or distort the development of high ability?”(p. 1). In the democratic South Africa it has never been an option to single out a group of learners with advanced abilities. This would just create another minority group, already privileged because of their special gifts (Taylor, Kokot & Heller, 2000). Moreover, this group of learners is often stereotyped as an elite group because of special education provision during the apartheid dispensation (Kokot, 1999).

South Africa has many mathematically gifted and talented learners – people who have an enhanced capacity to do Mathematics. They need to be better equipped for the challenges of a post-modern society and tertiary study, since far too many of the gifted currently do not stand even the remotest chance of achieving anything near their potential (Van der Westhuizen & Maree, 2006).

1.2.2 Policy context

The Constitution of South Africa, incorporating the Bill of Rights, informed the transformation in education. It stipulates that all individuals have the right to be respected and treated with equality and that every individual has the right to both basic and further education (RSA, 1996). Two other policies guiding education in South Africa are the White Paper on Education and Training of 1995 that outlined the principles of the new curriculum and the South African Schools Act No 84 of 1996 stipulating that high-quality education should be provided to all learners (DoE, 2001). The Education White Paper 6 answers the global call for inclusive education (DoE, 2001). Inclusive education in South Africa is in line with the Salamanca statement, signed in 1994, which shifted the focus for inclusion to the mainstream school and classroom (UNESCO, 1994).

Although education for the gifted has not specifically been mentioned in the White Paper, it nevertheless upholds the wider interpretation of inclusive education, namely the inclusion and support of all learners, and calls for respecting differences in learners and for enabling educational structures, systems and learning methodologies to meet the needs of all learners (Oswald & De Villiers, 2013). In a more recent document, *Guidelines for Inclusive Teaching and Learning* (DBE, 2010), the gifted learner is mentioned as one category of exceptionality when curriculum differentiation is discussed. The new curriculum initiative, the *Curriculum and Assessment Policy Statement* (CAPS) (DBE, 2011), identifies inclusivity as one of the general aims of the South African curriculum.

South African Grade 12 learners wrote their NSC examination based on the outcomes-based education (OBE) system for the first time in November 2008 and for the last time in 2013. The NSC curriculum was designed to embody the values, knowledge and skills envisaged in the constitution of the new democratic South Africa. It provided learners with the opportunity to perform at the maximum level of their potential and focused on high levels of knowledge and skills, while promoting positive values and attitudes (DoE, 2003). The NSC examined the extent to which a Grade 12 student had met the National Curriculum Statements (NCS) expectations as expressed in the Subject Assessment guidelines.

A learning outcome is a statement of an intended result of learning and teaching. It describes knowledge, skills and values that learners should acquire by the end of the FET band (Grades 10 to 12). Assessment standards are criteria that collectively describe what a learner should know and be able to demonstrate at the end of a specific grade. Such standards embody the knowledge, skills and values required to achieve the learning outcomes. Assessment standards within each learning outcome collectively show how conceptual progression occurs from grade to grade (DoE, 2003).

1.2.3 Organisational context: Stellenbosch University

The majority of the undergraduate students at SU are described as white and Afrikaans-speaking, with reasonably good schooling and from middle- to upper- class families, unlike most of their black counterparts. This is in a sense an “extreme case” in the post-

1994 era in the South African HE landscape, “a last bastion of white dominance” (Gibbon, 2010, p. 1). SU has a high level of success in terms of retention, throughput and graduation compared to most other African universities. This could be because most students enter the university with good secondary schooling, high family levels of education and good parental incomes. These are all factors that establish a good platform, from which students can benefit from the high-quality teaching and learning offered at the SU, and all the other systems and strategies in place to support them through their studies (Gibbon, 2010).

Notwithstanding all of this, the transition from school to university is still a period of new challenges and anxieties for many first-year students. Nel (2007) did a study on first-year students at SU and found that these challenges and anxieties occur not only on academic level, but also on personal and social levels. Students feel deprived of personal attention and they experience that nothing is compulsory any more, class sizes are very large and the workload higher and they must study for much longer hours. This is overwhelming for many students (Frick, 2008).

Factors contributing to students leaving SU without completing their studies were mostly academic failure (52%), financial grounds (17%) and combinations of the two (31%). Coloured students ranked financial difficulties highest, while language was a significant problem for African students. White students identified an active social life as the reason for failure. Most students (44%) who left SU did so after their first year of study. Staff and language diversity were also two factors seen as important for the improvement of retention (Nel, Kistner & Van der Merwe, 2013).

1.2.4 Professional context: The researcher's experience

The researcher has been teaching Mathematics for more than 20 years and started teaching the subject APM six years ago. She is currently teaching at a public school for boys. This school is privileged in terms of parental support, financial means and school resources. Mathematics is a popular subject and more than 92% of the boys take the subject. Many of these boys enrol at university after school.

University lecturers have taught the subject Additional Mathematics at this school since 2005 as an extramural activity. Since 2009 it has been the researcher's privilege to teach both Mathematics and Advanced Programme Mathematics. APM started with a small group of eight students, but it has become very popular, with a class of 32 in 2013. Many of these students give positive feedback about APM once at university, and claim that having taken APM made first-year Mathematics 'easy'.

As more and more learners are enrolling for APM, many parents, teachers and principals are enquiring about the merits of this subject. Parents, even if they cannot afford it, are prepared to go to any length to provide extra opportunities to prepare their children for success in their first year at university. As a Mathematics teacher, the researcher has often been involved in debates about university access tests and Mathematics marks as predictors of success in HE, but she noted that after the latest curriculum changes, parents and co-educators have become much more anxious about their children's transition from school to university. They want to know if their children should enrol in APM. This research wants to answer some of these questions.

1.3 PROBLEM STATEMENT

It has been observed that many intellectually talented learners following the NSC curriculum in South Africa are underprepared for university studies in Mathematics. Although these learners do obtain the required marks needed to gain access to university, they often lack the problem-solving, logical thinking, analysing and critical thinking skills necessary for the rigour of first-year Mathematics.

Many of these students never really studied much for Mathematics at school level and were used to obtaining high marks. They never really acquired the skills to work hard constantly and be disciplined in their studies, and few of them were ever challenged in any school Mathematics class. They enter the university with a perception that it is not necessary to work in Mathematics and that their marks should resemble their school Mathematics marks. Many of them are then disillusioned when they are not successful in Mathematics and are sometimes even at risk of failure. They experience their transition from school Mathematics to university Mathematics as traumatic, and some drop out in their first year.

There is a tendency in well-functioning schools to manage the learning programme tightly and to control learning. Teachers are reluctant to let learners ‘fall behind’ and go to great lengths, such as providing extra classes, to ensure that learners understand and master the curriculum, and are well prepared to do their best in the final NSC examination. Many of these students then never take responsibility for their own learning (IEB, n.d.).

Mathematically gifted students have unique learning characteristics and their abilities and motivation for academic achievement are well above the norm. Their specific intellectual needs are not always met in the current school system with the “one-size-fits-all” Mathematics approach offered in the current curriculum.

If students are unprepared in Mathematics when they enter university, it affects the quality of students graduating in the STEM fields. South African universities need to produce well-prepared STEM graduates with advanced knowledge and competencies for the challenging and changing demands of society and the economy. This is essential if South Africa wants to be a role player in the global economy.

This discontinuity between school and university or “gap” has a role player at each end of the gap, the university on the one side and the school system on the other. Universities cannot accept students for courses if they know in advance that they are underprepared and will most likely experience difficulties. What they can do, for example, is to raise their admission requirements, enrol more students in an extended programme, introduce more bridging programmes, change their curriculum or teaching strategies, extend their support programmes or do more research on the topic.

If higher education institutions are to remain relevant, that is, if we are serious about increasing access to higher education in an attempt to alleviate the critical shortage of science and engineering graduates in South Africa, we have to find ways to respond adequately to the problem of assisting under-prepared first-year students to succeed (Wolmarans, Smit, Collier-Reed & Leather, 2010, p. 10).

The school system, on the other hand, also has a crucial responsibility to help narrow the gap between school and university Mathematics. Possible options (among many) would

be to add an additional mathematics subject as another of the seven school subjects (Engelbrecht, Harding & Phiri, 2010) or to follow the global example of introducing Advanced Placement programmes.

Countries such as the USA, Canada and Australia have followed this route to strengthen students' tertiary readiness and numerous studies have been conducted that prove that there is a strong relationship between Advanced Placement (AP) courses and tertiary readiness and success measures (Casement, 2003; Dougherty et al., 2005; Geiser & Santelices, 2004).

The concept of an advanced course in Mathematics at school level is not completely new in South Africa. In the SC curriculum there was a subject, Additional Mathematics, which was removed with the implementation of the NSC Curriculum. Currently there is an APM course available as part of the curriculum of the Independent Examination Board (IEB).

APM is a subject defined by the curriculum document of the IEB as an extension of Mathematics and based on the same type of discipline. It aims to broaden learners' mathematical knowledge, deepen their mathematical thinking skills, and develop a passion for Mathematics and commitment to continued learning of Mathematics. It aims to enhance mathematical creativity and rigorous logical reasoning (IEB, 2006).

1.4 PURPOSE AND SIGNIFICANCE OF THE STUDY

The motivation of the IEB for the introduction of the APM to learners in any South African school, state or independent, reads as follows:

to provide interested and talented learners with the opportunity to study specific subject areas in greater depth, to provide schools with the opportunity to develop self-study skill in learners and ultimately to prepare good students for tertiary study (IEB, 2006, p. 3).

This study will aim to investigate the validity of these claims by evaluating the effectiveness of this APM programme in easing the transition from school to university Mathematics. To the author's knowledge very little has been published in South Africa

on this subject, and she hopes that this research will contribute to more knowledge production in the area of Mathematics and the transition between school and university.

1.5 RESEARCH QUESTION

To address the problems stated above, the following central research question guides and directs the study:

To what extent does the course, Advanced Programme Mathematics, prepare students for the rigour of first-year Mathematics in the STEM university programmes?

In order to answer this question, three secondary research questions are asked:

- *How are the APM and NSC Mathematics (Papers 1, 2 & 3) curricula related to the first-year Mathematics curricula (at Stellenbosch University)?*
- *To what extent, if any, do the learners who take the APM course and examinations prior to admission to the first year perform better in their first-year university examinations in Mathematics?*
- *What are the students' opinions on their experience of the effectiveness of APM in easing the transition between school and university Mathematics?*

1.6 OBJECTIVES

In order to undertake this study, the following objectives are pursued:

- To do a document analysis of curricula of the following courses:
 - National Senior Certificate Mathematics papers 1, 2 and 3
 - Advanced Programme Mathematics
 - First and second semester first-year Mathematics (at SU)
- To do an empirical investigation to determine the difference between the students who took APM and those who did not, when the following factors are compared:
 - Performance in the NSC examinations and performance in first semester examinations of first-year Mathematics
 - Performance in the NSC examinations and performance in second-semester examinations of first-year Mathematics

→ Performance in the NBT and performance in the first- and second-semester examinations of first-year Mathematics.

- To determine the relationship between APM marks, NBT marks and the first- and second-semester Mathematics marks.
- To determine, by means of a questionnaire, how the students experienced the effectiveness of APM in easing the transition between school and university Mathematics.

1.7 THE RESEARCH PROCESS

1.7.1 Research design and methodology

This research was designed according to a new framework introduced by Plowright (2011), referred to as the Framework for an Integrated Methodology (FralM). This framework proposes a fundamentally different way of thinking about, and doing educational research. It is a model that describes the process of designing, planning and carrying out research and aims at “supporting the integration of different elements of the research in the process of studying a topic, without favouring a certain element over any other” (Plowright, 2011, p. 4). The FralM is a combination of a “pragmatic integrated methodology, a relativist social epistemology, a realist social ontology and a realist object ontology” (Plowright, 2011, p. 186).

This means that this research will focus on the purpose, which is to determine the usefulness of the subject APM, and it will be driven by the research question and not the researcher’s own philosophical position prior to the beginning of the research.

Plowright refers to the philosophical perspective of this approach as “holistic integrationism” (Plowright, 2011, p. 186). It relies heavily on the paradigm of pragmatism, which argues that the truth is ‘what works’. This study therefore aims to provide “a well referenced, coherent and organised account, that draws on rigorous empirical research that enables the researcher to provide evidence to support any claims she has made, in order to answer the research question” (Plowright, personal communication, 22 October 2014).

1.7.2 The participants and research data

After taking into account the different contexts in which the research is planned and undertaken and formulating the research question, the task of data source management starts and the sampling decisions underlying the case selection are made. Then the methods of data collection to be used, the types of data to be collected and the analyses of those data are chosen (Plowright, 2011). According to the FraIM there are three methods of data generation and collection i.e. observation, asking questions and artefact analysis.

For the first part of this study, the artefacts analysed are the curricula of the NSC Mathematics Papers 1 and 2, the APM paper and the first and second semester papers of first-year Mathematics. In the second part of this study the participants or case studies were purposely selected from the 2013 cohort of first-year students at the US who enrolled for either Mathematics 114 and 144 or Mathematics 115 and Mathematics 145. They were divided into two groups: those who had taken APM in high school and those who had not. Apart from the demographical data of these students, other sets of data of students used this stage of this study are their marks in the NSC Grade 12 Mathematics Papers 1, 2 and 3 examinations, the NBT Mathematics examination, the APM examination, their Grade 12 average and their Mathematics 114 and 115 examinations or Mathematics 115 and 145 examinations marks. The third part of the study uses data obtained by means of a questionnaire sent to the target group,

1.7.3 Analysis of the data

The analysis of the data entails three parts. First, a document analysis is done to compare the different curricula in terms of content and cognitive levels and to provide the context for the empirical analysis that follows. Then analyses and interpretations of raw statistical data of the findings on the academic scores of two groups of students, those that took APM and those that did not, are done. In the third part, the perceptions of the students as stated in the open question of the questionnaire are analysed and discussed. The aim is to integrate the different parts of the analyses to present an integrated picture of the effectiveness of APM.

1.8 BRIEF CHAPTER OVERVIEW

Chapter Two presents the story of APM, as found in the literature. It defines the concept of an Advanced Placement Course and describes the development thereof over the last 50 years in countries all over the world. It aims to provide a critical discussion on the benefits of Advanced Placement.

Chapter Three discusses the researcher's journey in search of a theoretical framework for this study. It starts by giving background information on different types of theoretical frameworks and theories on Mathematical learning, and then gives some perspectives on curricula and thinking skills, after which it elaborates on the notion of student success and more specifically cognitive and non-cognitive predictors of success in Mathematics. Then the self-efficacy theory of Bandura, the self-confidence theory of Stankov and Ackerman, Kanfer and Beier's theory on AP are discussed as theoretical basis for this study, providing a conceptual framework to link the different theories.

Chapter Four starts by discussing the traditional concepts of 'paradigms' and 'designs' and then positions this study within the FraIM model of Plowright, which explains how this research will be conducted. The sampling method, the data- collection procedures, the instruments used, the data presentation and the analysis procedures used are discussed for each of the three methods used in the study.

Chapter Five uses document analysis, an empirical analysis and a narrative analysis to analyse the data and then integrates the analyses of the different methods to answer the research question.

Chapter Six presents the final discussions and findings of this study, as well as recommendations for students, schools, teachers and curriculum planners. The limitations of this study and recommendations for future research are presented and conclude this investigation. References will follow.

This study should be seen as only a first attempt to do justice to the subject APM as possible instrument in easing the transition from school to university Mathematics.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The transition from high school to higher education (HE) is a complex and broad topic. It can be seen as a bridge that students have to cross between two worlds. This bridge consists of many different parts, such as the preparedness of the students, the admission process and entrance requirements, and subsequent academic success. Building the bridge involves many different role-players, starting with the school sector, the policymakers, and then HE and the educational researchers. One of the gatekeepers at this bridge is Mathematics. Currently this gatekeeper is seriously ill, as there is a big gap between school and university Mathematics. This gap can only be mended when considering all the different building blocks involved in building it. If mended, many mathematically gifted students will be attracted and will cross the bridge on their way to degrees in Science, Technology, Engineering and Mathematics (STEM).

University entrance systems vary widely from country to country and from institution to institution. In order to prepare students for this transition, many different types of pathways have been developed. These pathways include bridging courses and access programmes that are followed after completion of high school, as well as advanced academic programmes, comprising university level courses and examinations taken while in high school. Some of these programmes only prepare students for participation in HE, while others aim to prepare them for academic success in HE.

For the purpose of this study, the emphasis will be on the university-level academic programmes followed in high school *in addition* to the normal high school syllabus offerings, specifically those in Mathematics. All the courses in these programmes are referred to as Advanced Placement (AP) courses and focus on students whose ages range from 17 to 19 years in the last two years of high school. The term 'Advanced Placement' refers to the fact that many participating colleges and universities in the

USA, and more than 30 countries around the world, grant credit and/or AP to students who do well in the examinations. This chapter will begin with a review of the literature available on these courses worldwide, and then specifically in South Africa.

As an introduction, this chapter will first describe the different formats of AP courses all over the world and then give a brief discussion of the origins, development of, and philosophy behind these courses. Then the critical questions asked and the issues encountered concerning AP will be reviewed and made operational in the South African context. The main emphasis of this chapter will be on analysing literature on the appropriateness of the AP Mathematics courses in preparing intellectually gifted learners for university Mathematics. The chapter will conclude with a summary of the possible benefits of APM in bridging the gap between school and university Mathematics.

2.2 THE CONCEPT OF AN ADVANCED PLACEMENT COURSE

Advanced placement is a form of subject-based acceleration, which allows for the introduction of advanced content earlier than customary. Pressey's (1949) definition describes acceleration as "progress through an educational program at rates faster or at ages younger than conventional" (p. 2). According to this definition, Southern, Jones, and Stanley (1993) identified 17 educational types of accelerative options. Advanced placement is only one of these options. Some of the other options include grade skipping, early admission to first grade, self-paced instruction, curriculum compacting, telescoping curriculum compacting, mentoring, extracurricular programmes and correspondence courses.

Advanced placement opportunities are viewed as one the most effective and comprehensive mechanisms in place for meeting the educational needs of students whose abilities and motivation for academic achievement are well above the norm (Lubinski, 2004).

The AP programme consists of courses and examinations that introduce high school learners to university level core content, knowledge and skills. The aim is to develop their critical thinking skills by promoting inquiry and problem solving, often in an

interdisciplinary mode. The final assessments are designed to measure how the students apply the knowledge and skills gained through the course (Mattern, Xiong & Shaw, 2009).

2.2.1 AP courses in the USA

The concept of an 'AP course' originated in the USA. The first AP course was introduced in the USA more than 60 years ago in the 1950's, shortly after the Second World War. It was one of the most successful professional initiatives that resulted in the institutionalisation of acceleration. Started by the Ford Foundation for the Advancement of Education, it originally involved three private high schools and three universities (Harvard, Yale and Princeton) to design a set of achievement examinations to give bright, hardworking students AP at university (Nugent & Karnes, 2002).

The College Board took the responsibility for administering the programme in 1955. This same organisation runs the Scholastic Aptitude Tests, a test for readiness for universities in the United States (US). Today AP examinations are one of the largest assessment programmes in the world and a widely known component of the US educational system. The AP programme in 2014 includes 37 examinations in 22 subjects administered to over 1.5 million students. Each examination has associated instructional support material. These courses and examinations are delivered to almost 18 000 schools by over 120 000 teachers and are offered in 115 countries (College Board, 2014)

The AP programme is taught by schools, which pick subjects in their areas of strength and interest, and then allow their students to pick courses from the ones that are taught in their school. These courses are provided free to the school by the College Board. Students also have the flexibility to study independently or online and can then write the AP examinations at the nearest test centre (College Board, 2014).

Students may choose to take AP courses and examinations any time during high school, but the majority of students wait until 11th and/or 12th grade. There is no required number of courses – no minimum or maximum. AP courses are one year in length and examinations are taken in May. AP examinations take two to three hours and the

students' AP grade is determined solely by their performance in the examination in May. Their scores, graded between '1' (indicating below average understanding) and '5' (indicating complete comprehension) determine whether they can earn advanced credit standing in undergraduate university programmes. A score of 3 or more generally qualifies a student for university credit. Students can take an AP examination without taking an AP course (College Board, 2014). The structure of the AP programme in the USA provides a mechanism by which students who engaged in accelerated learning in high school might bypass previously mastered material once at university (Mattern, Xiong & Shaw, 2009).

The available AP courses with mathematical content are Advanced Placement Calculus and Advanced Placement Statistics. Within the AP Calculus programme there are two courses and examinations. The AB Calculus course covers the content that is typically taught in first semester university calculus, while the BC Calculus course covers content typically taught in the second semester of university Calculus. In 2004, over 225 000 students took an AP Calculus examination, with over 50 000 of these students taking the BC examination. Since the first examination in 1955 the AP Calculus programme has grown at a rate of 7-8% per year (Bressoud, 2004).

2.2.2 Other international courses with curricula similar to those of AP courses

An acceleration programme similar to the AP Programme in the US is the Cambridge A-Levels course. The Cambridge A-Levels is a flexible two-year course with a choice of 55 subjects that schools can offer in almost any combination. This flexibility means that schools build an individualised curriculum and learners can choose to specialise in a particular subject area or study a range of subjects. It is typically for learners aged 16 to 19 years who need advanced study to prepare for university and HE. Learners use Cambridge International AS and A Levels to gain places at universities. The Lisbon Convention (an international agreement signed by 50 countries and international organisations, including the European Union, USA, Australia, Canada, Israel and New Zealand) facilitates the recognition of foreign studies among the signatory countries.

It should be noted that Cambridge International A Levels are different in structure from UK A Levels, which is the school-leaving qualification offered by educational bodies in

the United Kingdom and the British Crown dependencies, to students completing secondary or pre-university education. Whereas UK A and AS Levels are modular (students can retake individual components at the end of each year) the Cambridge International A Levels have a linear structure with examinations at the end of the course, which encourages more integrated study of the entire subject. Cambridge International Examinations is the world's largest provider of international education programmes and qualifications for 5- to 19-year-olds. They are a division of Cambridge Assessment, a department of the University of Cambridge and a non-profitable organisation.

Cambridge A Level testifies not only to the student's content knowledge in a subject, but also to his/her ability to present a well-reasoned argument, to understand and apply principles and to acquire deep understanding of a body of knowledge. Cambridge A Levels are taken in more than 125 countries, with 350 000 entries each year. The A Level courses in Mathematics offered are Mathematics and Further Mathematics (Cambridge International Examinations, 2014).

Another programme that fits the description of an AP programme is the International Baccalaureate (IB) diploma programme. The IB is an international educational foundation based in Geneva, Switzerland, that offers the International Baccalaureate Diploma Programme (IBDP). It is a two-year programme for students aged 16–19. IBDP students complete assessments in six subjects from six different subject groups and three core requirements. Currently the IB provides educational programmes in over 800 schools throughout the world. These are offered to students whose ages range from 17 to 19 years in the last two years of high school. Although the first IB schools were predominantly private international schools, today over half of all IB schools in the world are state schools. This course offers four Mathematics subjects; these are offered as standard-level courses of 150 hours or higher-level courses of 240 hours over the two-year period (International Baccalaureate Diploma, 2014).

In the curriculum of the Australian course, Specialist Mathematics compares well to the AP course. Courses comparable to AP in Canada are Pure Mathematics 30, Applied

Mathematics 30 and Mathematics 31, developed and examined by Alberta's Department of Education (Sproule, n.d.).

A number of countries in southern Africa, including Botswana, Swaziland, Uganda, and Kenya, offer APM courses. Most of these emerging market countries use existing advanced Mathematics programmes from other nations or internationally accredited agencies, of which the Cambridge International Levels are the most popular (Sproule, n.d.).

2.2.3 AP courses in South Africa

In South Africa, there is no course which is referred to as an 'Advanced Placement course'. However, the concept of an AP course, as discussed above, is similar to the Advanced Programme Course in South Africa. These courses were developed by the Independent Examination Board (IEB), and currently only comprise three subjects, i.e. Mathematics, English and Afrikaans. The Advanced Programme Mathematics (APM) course was introduced in 2008, Advanced Programme English in 2012, and Advanced Programme Afrikaans in 2013. Some of the content of the APM course resembles that of Additional Mathematics, a subject that was part of the South African Mathematics curriculum before 2008.

The Advanced Programme courses are available in both independent and public schools in South Africa. In the independent schools these courses are an elective of the IEB syllabus, offered in addition to the school syllabus, while in state schools they are offered after school as an extramural activity for a maximum of two hours a week. The school provides the teacher and the learners pay to write the Advanced Programme examination, for which they receive an additional certificate (Independent Examination Board [IEB], 2012).

In this study the focus is on Mathematics, and from now on any reference to a South African Advanced Programme course will refer to APM.

2.3 THE HISTORY OF AP PROGRAMMES IN THE USA

In the 1950's the AP programme was seen as a way of engaging and challenging high-achieving students at the best high schools in the USA. School administrators and reformers concerned with these students argued for increased rigour and tracking students by ability. They believed that the advanced levels of work could only be done in exceptional secondary schools, public as well as independent, and that able students could push themselves harder, and by means of extra courses and summer work achieve earlier graduation and entrance into graduate school or the workplace. High school students could earn credit, or at least AP, for university level coursework. In this way they could avoid needless repetition once they arrived at university. In 1954, only 532 students in 18 participating schools took 929 placement examinations (Bailyn, 1972).

The 'best and the brightest' had to be challenged and subsequently meet the intellectual demands of scientific and political leadership positions in a Cold War world. Dudley, the second director of AP, wrote "the basic philosophy of the Advanced Placement Program is simply that all students are not equal" (Schneider, 2009, p. 1).

In 1957 the launching of Sputnik raised questions about the ability of the USA to compete intellectually and scientifically against the Soviet Union. In this context the idea of providing an edge to US students in programmes such as AP seemed appealing to professional educators. Because only a few schools were involved initially, and only the highest achieving students were invited to participate, AP quickly became a mark of academic prestige. Unintentionally, the creation of AP would be the creation of "a 'branded' curricular status symbol" (Schneider, 2009, p. 816). Teachers embraced the opportunity to teach AP and in doing so work with able students, challenging material and in a heterogeneous class. In these early years, some of the pedagogical concerns already were that this programme dramatically increased the workloads and stress levels of the academically gifted child and the teacher (Hampel, 1986).

The late 1950's saw increased applications for a fixed number of universities and university places and pressures for admission building up. AP was becoming a standard part of an impressive résumé, regardless of the degree to which it actually prepared

students academically (Schneider, 2009). In the early 1960's AP programmes were still reserved for wealthy independent schools and high schools in affluent suburbs. The elitist nature of the programme was recognised but rarely questioned (Bragdon, 1960).

It was only by the late 1960's that growing equity-based criticism of AP started. AP was recognised as a haven for white, upper-class students, and this was furthering inequality and touching on "institutional racism" (Schneider, 2009, p. 820). Responding to this criticism would lead to the programme's rapid expansion. The democratic trends of the late 1960's and early 1970's in the USA called for "better education for the many, rather than the best education for all" (Rothschild, 1999, p. 185). The emphasis shifted from the education of gifted children to equity issues. Although AP provided many opportunities, from advanced study to advantage in admissions, it was only for those who had access to it. At this stage AP students were primarily white and lived in suburbs or attended private schools. White children in rural areas and black children, who were largely confined to less-affluent urban schools, had no access to AP programmes (Hochman, 1970).

In the mid-1970's, AP became a lever for school reform in underserved communities. Schools with AP, with the help of the College Board, promoted AP and schools without AP continued to adopt these programmes. Throughout the 1980's the programme remained a way for newly established schools to gain credibility with both parents and universities (Schneider, 2009).

The 1980's and 1990's saw the rapid expansion of AP. In 1986, the number of students taking AP were 231 000 in 7 201 schools and in 1994 there were 458 945 students in 11 500 schools enrolled for AP. Although this was still only a small fraction of US high school students, there had been a big increase in numbers among students of colour (Wiley, 1989). By 1994 minorities accounted for 26% of those who wrote the AP tests. Not only individual schools and local school districts realised the usefulness of the programme, but also the US federal government. In 1990, they spent 2.7 million dollars subsidising AP examination fees and teacher development in low-income districts (Schneider, 2009).

By the dawn of the 21st century, AP had expanded from private and suburban public schools to schools serving a high percentage of minority and low-income populations. From 1990 to 2000, the high schools participating in AP programmes increased by 40%. This expansion meant greater opportunities for everyone, but the level of prestige and achievement associated with the programme declined (Schneider, 2009). The elite status of AP dropped and nearly every able and ambitious US student had access to AP. More and more school districts dropped their entrance requirements and encouraged students to take rigorous courses. This decision was often framed in terms of access, equity and university readiness. Some state policy makers began mandating the inclusion of AP courses in their districts and high schools. As AP expanded, its prestige declined and many schools started to re-evaluate their AP programmes and pursue alternatives to AP. As AP lost its uniqueness and was adopted by a wider range of schools, it became more and more controversial (Sadler, Sonnert, Tai & Klopfenstein, 2010; Schneider, 2009; Wakelyn, 2009).

The story of the development of AP in the USA is an integral part of the educational history of the USA and more specifically of curriculum reform in the USA. It highlights the fact that very often in educational reforms political concerns prevail over educational and pedagogical concerns in order to mediate conflicting interests in the political domain (Cross, Mungadi & Rouhani, 2002). Just as in the development of AP in the USA, the following competing agendas have also been proposed for basic education in South Africa: cultural diversity, national unity, 'liberation', widening access, equality and excellence. It can therefore be very appropriate for a country such as South Africa to 'borrow' educationally from another country, in this case the USA. Jonathan (2000, p. 3) describes this opportunity:

look at what has been tried in one place and think what relevance it might or might not have elsewhere. But there is another type of opportunity, which regrettably, is less often taken. That is to share our understanding from one society to another, not so much of what seems to work, given adequately favourable conditions, but of what has not worked anywhere, whatever the policies implemented to bring it about.

It was therefore necessary, as Jonathan argued, to discuss the issues of development of AP in the USA in order to gain a better perspective on the development of such a course in South Africa.

2.4 THE ADVANCED PROGRAMME MATHEMATICS COURSE IN SOUTH AFRICA

The APM course is a three-year acceleration course aimed at learners in South Africa who have a greater than average ability in, or enthusiasm for Mathematics, and who intend to pursue careers in STEM fields. The syllabus covers topics similar to both school and university Mathematics. The Grade 10 syllabus consists of five learning outcomes, i.e. Calculus, Algebra, Statistics, Mathematical Modelling and Matrices and Graph Theory. In Grades 11-12 the topics are Calculus and Algebra and then the third topic is a choice of Statistics, Mathematical Modelling and Matrices and Graph Theory (DoE, 2006).

The APM course is taken in addition to the normal school Mathematics course and is presented on a lecture-tutorial model. Instruction time ranges from one hour in Grade 10 to two hours per week in Grade 12. Students are required to organise their own study groups to discuss and understand problems, with teachers making direct interventions on a limited scale. Teachers aim to develop in students the ability to identify when they need assistance and how to seek that assistance.

The motivation of the IEB for the introduction of the AP to learners in any South African school, state or independent, is

- to provide interested and/or talented students with the opportunity to study specific subject areas in greater depth;
- to provide schools with an opportunity to develop self-study skills in learners; and
- ultimately, to better prepare good students for tertiary study (IEB, 2006).

APM aims to broaden the learners' mathematical knowledge, deepen their mathematical thinking skills and develop a passion for Mathematics and commitment to continued learning of Mathematics. APM also aims to contribute to the personal

development of learners by providing challenging learning experiences and feelings of success and self-worth. It aims to develop appropriate values and attitudes and further the appreciation of the development of Mathematics over time, establishing a greater understanding of its origins in culture and as a tool in society (IEB, 2006).

2.5 THE HISTORY OF APM IN SOUTH AFRICA

APM as it is currently known originated in 2006 during the implementation of a new curriculum in South Africa. Because a curriculum is a product and expression of the political interests, values and knowledge of the dominant social group, curriculum knowledge cannot be neutral (Jansen, 1998). Therefore, to position APM within the South African Mathematics education scene, it is necessary to give a brief overview of curriculum development in South Africa in the post-apartheid era before discussing the development of APM.

2.5.1 Curriculum development in South Africa in the post-apartheid era

South Africa is a young democracy of two decades within which four waves of curriculum reforms have occurred. The first attempt was in 1995 with the White Paper on Education and Training that called for the transformation of the school curriculum and formation of democratic structures (DoE, 1995). Jansen (1999) describes this as a sort of ‘cleansing’ through cutting out offensive and outdated aspects of the apartheid curriculum, regardless of their pedagogical soundness.

In 1997, a second educational approach, called Curriculum 2005, was announced. This new Outcomes Based Education (OBE) was part of a global educational curriculum reform phenomenon and brought about radical reform in school curricula. It brought a change from a content-based curriculum to a curriculum that specified different outcomes as learning goals (Cross, Mungadi & Rouhani, 2002). It also introduced a change from ‘fundamental pedagogies’ (a racially based prescribed set of learning objectives) to ‘progressive pedagogy’ (learner-centred teaching and learning strategies) (Cross et al., 2002, p. 10). These outcomes are demonstrations of what learners can do with their knowledge, rather than the production of knowledge (Rogan, 2007). Cross et al., (2002) argue that underpinning Curriculum 2005 is the integration of education and

training (as opposed to the separation between academic and vocational education in the old curriculum), the need for an integrated regulatory framework which gained form in the National Qualifications Framework, and concerns with the job placement needs of learners in the context of globalisation. It can also be linked to the debates on the changing mode of knowledge production.

The National Curriculum Statement Grades 10 – 12 (General) seeks to promote human rights, inclusivity, environmental and social justice. In particular, the National Curriculum Statement Grades 10 – 12 (General) is sensitive to issues of diversity such as poverty, inequality and race (DoE, 2003).

Many educationists in South Africa have taken a stance either for or against OBE. Some scholars, such as Mahomed (1999) and Malcolm (1999), have defended OBE while others, like Jansen and Christie (1999), Chisholm (2003) and Jonathan (2000), have been very sceptical.

In 2008 the National Senior Certificate (NSC) replaced the Senior Certificate (SC). The distinction between higher grade and standard grade was abolished, and part of the Mathematics syllabus (Euclidean Geometry, Probability, and Recursive Patterns) was moved to an optional third paper. The 2008 school-leavers were the first group of learners who had followed the OBE curriculum for their entire school career.

In 2012 the Curriculum and Assessment Policy Statements (CAPS) was released, to be implemented from 2012 to 2014 (DBE, 2011). This document identified inclusivity as one of the general aims of the South African curriculum. Inclusivity is therefore foregrounded, and giftedness is identified as one of the 'exceptionalities' that need to be addressed (Oswald & De Villiers, 2013). Although the research in this study was done on OBE learners, this fact is positive for the future of education of the gifted in South Africa.

2.5.2 The development of the APM course

With the implementation of the National Curriculum Statement in 2006, one of the subjects, Additional Mathematics, was removed. This was an optional subject in the syllabus, aimed at mathematically talented students, which laid a solid foundation for university studies in Mathematics. Other major changes were the reduction of the

number of subjects for Grade 10 students and integration of higher grade and standard grade Mathematics into one Mathematics course.

Subsequent to these changes, a group of passionate teachers, from both independent and state schools, and university mathematicians, under the leadership of Stephen Sproule, decided to correct what they saw as a problem. They had worked since 2004 to prepare 17 unit standards for a new subject called APM to replace Additional Mathematics. The new NCS document opened the door for this, by stating that a learner could take a maximum of one subject from any other assessment body (with the permission of the Minister of Education). These unit standards were registered by the IEB (Independent Examination Board) at the South African Qualifications Authority, and their quality assured by the Council for Quality Assurance in General and Further Training (UMALUSI).

The IEB was not the only organisation that introduced an APM course. Several private tutors started to establish centres throughout the country where the APM syllabus or a large part thereof was taught. One of these centres, the Admaths Centre in Brackenfell, also developed an online course for APM. Other examples of such centres are Advantage Maths in KwaZulu-Natal and Alpha Mathematics in Gauteng.

In the APM curriculum statement (DoE, 2006), the IEB makes several claims regarding the benefits of APM. It claims that the study of APM will enable learners to a) extend their mathematical knowledge and develop confidence in their ability to solve new problems, b) solve these problems with sophisticated mathematical processes and in a creative and critical way, c) persevere patiently when problems require more time, both on their own or with peers, d) validate their own answers and to focus on the process and not just on the right answer, e) use the language and symbols of Mathematics to communicate, f) be able to see the relevance of Mathematics in their everyday lives and Science and to use these effectively as problem solvers in their communities.

To the author's knowledge, no literature is available on South African AP courses that can confirm or refute these claims. It was therefore necessary to engage with available literature from other countries to determine the impact of AP courses and the relevance of these claims.

2.6 GLOBAL CONCERNS ABOUT AP PROGRAMMES CONCEPTUALISED IN SOUTH AFRICAN CONTEXT

Volumes of articles and books commenting on the AP programmes in the USA have been published over the last 50 years. Given the South African situation where the AP programme is still in its early developmental stage, it is important to take note of the concerns raised about AP courses in the USA.

Strydom, Kuh and Mentz (2010) developed a table summarising the similarities in challenges between the USA and South African HE contexts. They base their work on an analysis of research by Kuh (2007) and Scott (2007). They highlight that addressing these challenges in the specific context is difficult, as the magnitude of the challenges are clear, given the socio-economic, capacity, and resource constraints faced by South Africa as a developing country. Table 2.1 summarises the similarities of the challenges facing HE in South Africa and the USA.

Table 2.1: Comparison of challenges facing higher education in USA and South Africa

UNITED STATES OF AMERICA	SOUTH AFRICA
Low pass rates	Very low pass rates (about 15% graduate in time)
Low enrolment of minority group students	Participation rates of previously excluded Black African students around 12%
Lower pass rates amongst low income, minority group students	Only one in three Black African students graduate in time, less than 5% of this cohort obtains a degree.
Students not adequately prepared in high school	Students not adequately prepared in high school
Increased demand for graduates in the knowledge economy results in a rapidly expanding student body with unprecedented levels of diversity and large numbers of first generation students	Widening access and an increased demand for graduates in the knowledge economy lead to unprecedented levels of diversity and many first generation students.

Source: Strydom, Kuh and Mentz (2010)

The concerns raised about AP programmes in the USA can be categorised as curriculum issues, increased access issues and equity issues, gender issues, financial issues, and then the purported benefits (non-academic and academic) of AP. For the sake of brevity, only some of these categories will be discussed in brief in the next

section. The emphasis of this study is on the possible benefits of AP courses in Mathematics.

2.6.1 Curriculum issues

One of the criticisms against the AP programme is that the curriculum is often too much for the time-bound course, i.e. too many important topics have to be dealt with in too little time and in the process they lose their meaningfulness. It is then capped with a high-stakes, breadth-orientated (content) examination (Parker et al., 2011). Including too much accelerated content in AP courses can prevent students from achieving deep contextual understanding of the content and the unifying concepts of a discipline. The AP course must rather focus on the development of skills of inquiry, analysis and problem solving (Bleske-Rechek, Lubinski & Benbow, 2004).

Other scholars argue that AP classes do provide students with appropriate and challenging curricula by allowing them to study advanced material at a pace commensurate with their advanced rate of learning (Bleske-Rechek et al., 2004).

The author has experienced that in South Africa, when new content that is not part of the NSC syllabus is introduced in an APM class, the gifted learners perceived it as intellectually stimulating and challenging. This is in line with Lubinski's (2004) research that argues that the distinct learning preferences of gifted children necessitate a differentiated curriculum and that acceleration options address the gifted student's capacity to learn more quickly and with less direction from the teacher.

With good planning and one to two hours tuition a week it is possible to complete the syllabus in three years from Grades 10 to 12. However, it is necessary that the students take responsibility for their own work, are willing, and have the time to work independently on their assignments for several hours a week. They must be able to check their own work against those of their peers and curious to discuss possible solutions to problems. The author is of the opinion that the examinations are challenging and require inquiry, problem-solving skills and definitely the ability to work against time. These skills are all keys to success at HE level.

2.6.2 Issues related to increased access

A large volume of research shows that even when access to AP courses in the USA increased rapidly, the minorities and students from low socio-economic levels in AP classes were still underrepresented (Hertberg-Davis, Callahan & Kyburg, 2006; Klopfenstein, 2004; Moore & Slate, 2008; Ndura, Robinson & Ochs, 2003; Whiting & Ford, 2006). These are the students who could benefit most from the study skills, writing skills and university preparation that AP classes offer (Vaughn, 2010). Participation in AP classes has proven to be an important factor in the future financial success and job marketability of these students (Flowers, 2008).

One reason for the underrepresentation of minority students and students from low socio-economic levels in AP courses is not their lack of ability, but rather their teachers' hesitation to identify giftedness in students who are culturally diverse or who come from low socio-economic backgrounds (Ford, Grantham, & Whiting, 2008; Gallagher, 2005; Miller, 2009; Yoon & Gentry, 2009). Buice (2012) gives another reason. These students are not aware of their potential, do not believe in themselves and do not get any encouragement from their communities. They follow what they see and they see lack of support for rigorous classes. In the African American community, the community sometimes downplays students acting "white". If these groups of students knew that they had the ability to succeed and received the necessary encouragement, there would be no limit to their potential.

The expansion of the AP programme into contexts where students are not ready, or to schools without sufficient resources for supporting student success programmes, is often counterproductive. The same is true when taking the AP course and not studying for the examination (Sadler et al., 2010).

Klopfenstein (2004) states that low income reduces AP participation by 40% for all races, but that black and Hispanic students are three times more likely to come from low-income backgrounds than white students. If these AP students do attend university, they will be the ones more likely to receive scholarships. Thirty-one percent of universities look at participation in AP courses when determining scholarship awards (College Board, 2014).

Many of the minority students are first-generation immigrants who often work to help support their families. Enabling these students to attend a post-secondary school could be the beginning of a future in a new country. AP classes may help to bridge the gap between the two populations, and schools must look at ways to ensure that all students receive an equal chance to excel in the classes that will best prepare them for their future (Buice, 2012). In South Africa, the first-generation students are not immigrants. They form the majority of the people in the country (Van der Berg, 2007).

It is clear that educational disadvantage and opportunity manifest themselves in the gatekeeping subject of Mathematics, especially at Grade 12 level (Vithal & Alant, 2005). The question of how and why Mathematics acts as a filter of equity is a longstanding one, as seen in the magnitude of current literature on Mathematics education (Adler & Setati, 2005). Vithal and Alant (2005, p. 10) describe the frustration of the learners of Mathematics in South Africa in the following words:

We still observe a sanitised voice, a mediated voice, and a rupture between that which we read in our research, policy and theory volumes and the anger, frustration, emotion and passion with which learners speak, especially those for whom the system fails. The challenge of how to capture, represent and theorise learners' authentic voice and experience in mathematics and science education remains, as does questions of the place of mathematics and science education in the pathways of education linked to power and status in the twenty first century.

There are volumes of writings on the different aspects of schooling that have been obstacles in the achievement of equity in the South African schooling system over the past thirty years. Most of these writings argue that the history and legacy of inequality education in South Africa and the disparities in terms of social class, language, gender, race and location have led to the inequalities. These inequalities include issues like resources, teacher provision, teacher expectations and behaviours, financial resources , to mention a few (Sayed, Kanjee & Nkomo, 2013). Holborn (2013) makes the arguable statement that the failures in South Africa's education system reflect the problems in governance in the country since 1994. She names a lack of skills, monitoring and accountability as factors that led to poor policy implementation, inferior training of

teachers and bureaucrats and a system many people have lost hope in. Xolo (2007), himself a former disadvantaged gifted South African, argues that if one is to help the gifted disadvantaged, the first thing to do would be to identify them correctly, that is, to be able to see the potential in the learner. Thereafter, it is necessary to help them see that they are gifted and to encourage them to work even harder so that they can maximise their potential. He stresses that motivation is the best way to help a disadvantaged learner to focus on the positive aspects about himself and his immediate environment, convincing him that he can do it no matter how contrary the circumstances may be.

The problem with most intervention programmes is that they are designed for a few privileged communities and that the gifted disadvantaged are often left to fend for themselves. “We must now take not only people to the universities, but also the universities to the people, especially the disadvantaged people” (Xolo, 2007, p. 3). Hertberg-Davis and others (2006) advocate the modification of instructional strategies to provide the supportive framework necessary for the success of a broad range of students.

2.6.3 The financial cost aspect of AP programmes

Several authors raise concerns associated with the cost of supporting AP programmes, primarily the cost of the AP examinations themselves. In 1999, the US Department of Education provided 4 million dollars in grants to pay examination fees for financially disadvantaged students enrolled in AP programmes (Curry, MacDonald & Morgan, 1999). Taxpayers ultimately subsidised these costs. These concerns became particularly important, as some states had recently started planning to mandate the availability of AP courses (Sadler et al., 2010).

Klopfenstein and Thomas (2010) warn that expanded AP enrolment has severe financial implications for other curricula or programmes. Additional resources are flowing into AP courses at the cost of other high schools programmes. For example, the best and most experienced teachers are assigned to small AP classes, while non-AP classes necessarily grow larger. Teachers must be certified to teach an AP class, which entails completing an AP training workshop and the purchasing of AP training

materials. Some schools pay the AP examination fees for low-income students, using their state or federal government subsidies.

For South Africans in state schools, there are currently several options for a learner to take the APM course. The state schools that can provide a teacher with sufficient knowledge currently offer APM for free as an extramural activity in the afternoons. Any school in South Africa can enrol their gifted students for the APM course offered by the AdMaths centre in Bellville. Any individual can also enrol. This centre is a private initiative and charges a monthly fee. AdMaths reaches learners through classroom teaching as well as distance learning. The distance course is designed for self-study. Course material is available in Afrikaans and English and can be downloaded from the Admaths website in PDF format. It is compulsory to acquire a textbook Xolo (2007) urges private companies to contribute by sponsoring projects aimed at developing disadvantaged students in specific communities. The APM distance course is a viable project for such a company.

From 2008 to 2012 the IEB granted funding for the examination registration fees for APM learners in state schools. However, since 2013 learners have been requested to pay their own fees (R675 in 2013). Provision is made for learners who qualify for an IEB subsidy (IEB, 2012).

2.6.4 AP courses as an indicator of a school's quality

In the public opinion, the number of AP courses that a school offers is often seen as a valid indicator of the quality of the school. This is not necessarily true, as AP courses and teachers can vary significantly from school to school. In some schools, students get high grades in courses but are unable to pass the examinations. This might be because the content and calibre of the course were watered down (Klopfenstein & Thomas, 2010).

When adding an AP programme to a high school, this may disadvantage other programmes and the students in these other programmes through redirection of limited resources. For example, AP classes often have smaller class sizes and more effective and experienced teachers than other classes. As a result, the inclusion of AP at a

school would result in at-risk students being placed in larger classes with a greater likelihood of having a less experienced teacher (Klopfenstein & Thomas, 2010).

This practice currently happens in some South African private schools where the NSC curriculum competes with the APM curriculum for teaching time in a very full school day.

2.6.5 Non-academic implications of enrolling for an AP course

In 2004 Bleske-Rechek, Lubinski and Benbow, revealed the findings of three decades of longitudinal data on the AP programme. The study focused on the subjective feelings and educational outcomes based on AP versus non-AP participation. Participants were first- and second-year graduate students of universities in the USA who had been part of the AP programme while in high school. They reported on their high school experience. Overall, participants valued academic and intellectual stimulation in high school and found the lack of it distressing. They regularly voiced positive reactions to working hard, being intellectually challenged and being with their intellectual peers. Participants' high level of intellectual engagement was underscored by their likes and dislikes as a function of AP involvement. Those who did AP more frequently expressed satisfaction (and less frequently expressed dissatisfaction) with the intellectual calibre of their high school experience.

Overall, intellectually talented youth embraced and placed a premium on intellectual challenge in high school. Students believe that they receive better treatment from their teachers and enjoy increased homogeneity in academic achievement and interest among fellow students in these courses. These findings were consistent with previous findings on AP participation and ability grouping (Cross, Coleman, & Steward, 1993; Foust, Hertberg-Davis & Callahan, 2009).

Disadvantages of participation in AP courses that students identified were feelings of difference and isolation from students who did not participate in these courses. This was linked to the amount of time they spent on the course. They felt that their peers perceived them as academically superior and this made them feel uncomfortable in their company. The academic homogeneity of the AP classroom environment seemed to increase the students' comfort while learning. Stress and fatigue seemed to be

conditions AP students willingly accepted as intrinsic to the academic paths they saw leading to successful futures (Foust et al., 2009).

Gifted students who are intellectually stimulated are on a path towards personal thriving and living a good life. The gifted students taking AP courses express greater global satisfaction with their lives than their peers do. They specifically report elevated levels of satisfaction in their achievements, immediate standard of living, personal safety and future security than peers. They express powerful feelings of general self-efficacy and high levels of trait seriousness, two constructs related to facilitating success (Boazman & Sayler, 2011). This topic will be elaborated on in Chapter 3.

2.6.6 The possible academic benefits of AP

From the literature, it is clear that before discussing the academic benefits of AP, it is necessary to distinguish between the value of AP with regard to mere participation in the course or examination versus actual performance in the examination. Three groups of learners are identified: those who participate in AP programmes but never write the examinations, those who participate in AP but are not successful in the examinations and then those who successfully participate and write the examinations. (Successfully here means obtaining at least 3 out of a possible 5).

A great deal of literature is available on the possible academic benefits of AP programmes. This literature can be categorised under the following headings: AP as an opportunity for gifted learners; AP and other academic outcomes in high school (Bottoms & Feagin, 1997; Mo, Yang, Calaway & Nickey, 2011); AP and HE admissions (McKillip & Rawls, 2013); AP and first year grade point mark (Dougherty, Mellor & Jian, 2006; Scott, Tolson, & Yi-Hsuan Lee, 2010); AP and retention to second year (Mattern et al., 2009); AP and university going rates (Chajewski, Mattern & Shaw, 2011); AP and graduation (Wakelyn, 2009). Some of these categories will be discussed in the following section.

(a) AP as an enrichment opportunity for gifted students

That gifted students' needs frequently go unmet in the general classroom has been the subject of much discussion in literature on education for the gifted (Colangelo,

Assouline & Gross, 2004; Geake & Gross, 2008). Academic environments most likely to lead to personal thriving for the gifted are those that slightly exceed the gifted individual's current levels of academic performance and allow for intrapersonal and interpersonal growth (Gross, 2002).

These findings are consistent with the findings of Baozman and Sayler (2011). They argue that high-ability students who are accelerated academically are not affected negatively by the acceleration and that the acceleration may actually improve their social and psychological adjustment.

Watters and Diezmann (1997) determined that the central characteristic of mathematically gifted students is their advanced capacity to reason either analytically or spatially. Analytically gifted students are generally fast and accurate workers, who are able to articulate their chain of reasoning. In contrast, spatially gifted students may underachieve in classrooms because of the typical emphasis on analytical tasks and may experience significant difficulty verbalising their reasoning.

The environment for a mathematically gifted child should provide opportunities for these students to develop the skills to become autonomous learners. When gifted students work on relatively easy tasks, they prefer to work independently or side by side with another student. However, when tasks are sufficiently challenging, gifted students prefer to work in a group in order to share knowledge and access a support network. Gifted students should also have opportunities to work with like-minded peers who share their interests and will challenge their ideas. This may occur in the regular classroom or through acceleration or enrichment (Watters & Diezmann, 1997). AP courses can provide this learning environment.

(b) The influence of AP participation on other academic outcomes in high school

Students who participate in AP classes develop independent study skills (Hertberg-Davis, 2008) and receive encouragement from an academically supportive peer group (Shiu, Kettler & Johnsen, 2009). Intellectually gifted students who experience educational acceleration in high school view their pre-university educational

experiences much more positively than their intellectual peers who were deprived of such experiences (Lubinski, 2004).

(c) AP participation and its influence on HE admissions

AP is playing an increasingly larger role in HE admission (Klopfenstein & Thomas, 2010). Until recently AP was not linked to HE admissions. It was only in 2002 that factors such as enrolment in AP courses and AP examination grades were added to the list of admission factors (McKillip & Rawls, 2013). There seems to be a sizeable amount of research showing that AP courses can influence admission decisions positively. Universities rather focus on the number of AP courses taken than AP examination scores (Shaw, Marini & Mattern, 2013). Adelman (2006) has done extensive work on the role of high school rigour on degree completion and has included AP as a variable representing high school course rigour.

Since AP students usually score higher in university entrance examinations, it gives them a competitive edge in the recruiting process (Flowers, 2008). It not only gives them acceptance into the university, but they are more generally scholarship recipients as well (Wakelyn, 2009).

(d) AP results as a predictor of success in HE studies

The signalling ability of AP results as a predictor of success in HE studies is the single most important topic in the discussions on AP programmes over the past 50 years. There is an ongoing debate between those who claim a direct relationship between participation in AP courses at school level and academic success at university level, and those who believe there is no causal relationship. The College Board, owner of the AP trademark, provides several rigorous studies, showing that participation in AP in high school is a strong indicator of university success (Casserly, 1986; Morgan & Crome, 1993; Willingham & Morris, 1986).

Curry and others (1999, p. 9) summarise the above-mentioned studies and describe 'the AP effect' as follows:

AP motivates students, places them in classrooms with teachers who want to and love to teach, and gives them the strongest preparation in a subject. As a result, AP students in university are then better prepared academically, more likely to specialize in majors with tougher grading standard, more likely to complete more course work, more likely to be superior in terms of leadership and significant accomplishments, more likely to graduate with a double major, twice as likely to go on to graduate or professional school.

An increasing number of independent researchers, studying the AP programme, doubt the above-mentioned research on the causal effect of AP on university success. They have found that failing to control for the students' non-AP curricular experience leads to positively biased AP coefficients. For example, students who take AP courses are often a self-selecting group, and are more likely than other high school students to have personal characteristics such as being motivated and hardworking, which makes them candidates for later success (Klopfenstein & Thomas, 2010). These students will perform well in later assessments, enrol and persist at university even if they did not take AP courses and examinations because they are more motivated and better prepared when entering university. Students who have access to the AP programme typically attend better schools in higher socio-economic status communities (Dougerty et al., 2006) and AP courses in their schools tend to be taught by the more experienced teachers; they will consequently perform better (Paek, Ponte, Sigel, Braun & Powers, 2005).

Another concern in studies on the causal effect of AP on university success is that many studies omit Mathematics-taking information. Mathematics is frequently shown to be a strong predictor of university success and much of the estimated AP effect on the average AP student is actually the result of non-AP coursework in Mathematics and Science (Rose & Betts, 2001; Sadler & Tai, 2007).

When trying to identify the unique impact of AP after student background variables such as family income, parental academic background, class rank test scores and high school quality have been controlled, Klopfenstein and Thomas (2005) find no conclusive evidence that in the case of the average student, AP experience has a casual effect on

early university success. Mattern and others (2009), and Murphy and Dodd (2009) support these findings.

In later research Klopfenstein and Thomas (2010), however, distinguish between prediction and causality. Their research finds that AP course-taking alone may be predictive of university success. They do warn that the signalling ability will be partially diminished if the AP expands to students with less ability and motivation. Stakeholders in the AP programme mistakenly believe that AP enhances human capital and necessarily develops study skills and discipline necessary to succeed at university. They attribute these findings to the rapid expansion of the programme, the fact that many schools under pressure are simply renaming existing courses “AP” and the fact that many AP examinations are criterion-referenced, rather than normative (Klopfenstein & Thomas, 2010). In criterion-based assessment students’ achievements are compared to clearly stated criteria for learning outcomes and are independent of the cohort assessment, while with normative assessment students’ marks are adjusted to fit a normal distribution.

For students who participate in AP courses and examinations, research shows a positive relationship to many HE outcomes, even after controlling for prior academic performance and student characteristics (Chajewski et al., 2011). AP course participation and examination performance have been positively linked to attending a four-year course at an institution (Bowen, Kurzweil, Chajewski et al., 2011; Tobin & Pichler, 2006). The AP average score is also a significant predictor of a six-year graduation rate (Dougherty et al., 2006).

Of all the cognitive variables related to first-year grade point average, the high school grade average, followed by AP average, had the strongest relationship (Keng & Dodd, 2008; Shaw et al., 2013). AP therefore influences retention to the second year positively (Mattern et al., 2009).

This fact was confirmed in a very recent study of 26 693 students over 10 years by Ackerman, Kanfer and Calderwood (2013). There is thus a very strong relationship between post-secondary success and AP examination performance, but students who do not receive credit tend to perform at a similar level to those students who did not

complete any AP examinations. Students with larger numbers of AP-based course credits were associated with higher Grade Point Average (GPA's) and completion of a greater number of higher-level courses instead of lower level courses. Such students graduated at a substantially higher rate and in fewer semesters of study.

The most important predictors of STEM major persistence were firstly receiving credit for AP Calculus, and secondly successful completion of three or more AP examinations in the STEM areas (Ackerman et al., 2013).

2.7 SUMMARY

As seen in the literature review in this chapter, students perceive AP as a very challenging subject in school, and the programme is generally perceived by teachers, administrators, universities, policymakers, and the public as a source of academic rigour and challenge in high school. However, it seems that the curriculum and teaching are focused on motivated students with a history of school success and do not meet the academic needs of all gifted high school students. This is quite true in the South African context. Limited recruiting practices lead to underrepresentation of minority students and students from low socio-economic backgrounds.

The AP curriculum provides an opportunity for stimulation of gifted learners. Students enjoy and value the opportunity to work with similarly motivated students.

From the available literature, it seems that APM course-taking and examination-writing are definitely predictors of post-secondary success. In the specific South African context, as far as the author knows, no literature is currently available on the academic and non-academic benefits of APM. This fact underscores the necessity of this study.

In the next chapter, a theoretical framework underpinning this study will be discussed.

CHAPTER 3

THEORETICAL FRAMEWORK

3.1 INTRODUCTION

Through many decades, experts in the field of Mathematics education have advocated the use of research frameworks, theories and philosophical foundations as crucial aspects to consider when engaging in research activities (Lester, 2005).

This chapter describes the researcher's quest to find a theoretical framework for this study. It was a difficult and challenging journey. This chapter first positions the study within the different phenomena in Mathematics education and justifies the choice of a type of framework. Then an overview is given of the theories on mathematical learning to provide a context in which to determine the theoretical perspectives underpinning this study. Given that this study focuses on student success in first-year Mathematics, an overview of some of the most prominent theories on 'student success' is given. This is followed by a discussion of Bandura's self-efficacy theory, Stankov's theory on self-confidence, Paul Ernest's theory on mathematical empowering and then Ackerman, Kanfer and Beier's theory on Advanced Programme Mathematics (APM) as domain knowledge. The chapter ends with a synthesis and a description of a conceptual framework for this study.

3.2 CHOOSING A RESEARCH FRAMEWORK

3.2.1 Phenomena in Mathematics education

Schoenfeld (2010) gives an overall picture of the field of Mathematics education by categorising the phenomena that research in Mathematics education sets out to explain. He points out that the more different concerns and classes of phenomena there are, the more different theoretical lenses, perspectives and methods there will be. He lists some of the fundamental concerns in Mathematics education, including epistemology (a study of knowledge), cognition (how the mind works), content understanding ('understanding'

a specific mathematical term); pedagogy (effective teaching methods and good standards); equity and social justice (policy issues) and teacher knowledge (teacher decision-making).

To deal effectively with this range of issues in Mathematics education, it is necessary to take into account where the research is done, seeing that the education systems in different countries vary considerably. The system in the USA, for example, operates in a very different way from more centralised European and Asian educational systems, where mandates from educational ministries play a very strong role in determining curricula. In the USA, the 50 states have largely independent educational systems and within them, the 15 000 school districts have varied degrees of autonomy in setting educational goals and standards (Schoenfeld, 2010). It is interesting to note that Europeans in general have been more explicit and reflective about issues of theory than have educationists in the USA (Kilpatrick, 2010).

The phenomena in the Mathematics education of students born and bred in the Confucian heritage perspective are not as widely known. Confucianism can be described as a worldview or social political doctrine with religious quantities. It is a way of life that historically influenced countries and cultures influenced the cultures of China, Taiwan, Korea, Japan, Vietnam and Singapore (Tu, 1998). What is known is that the Confucian culture places high value on education, encourages hard work and emphasises effort over ability. Parents have aspirations for their children and are highly involved in their children's schoolwork (Chen, Lee & Stevenson, 1996). Because of the differences in cultures, it is often difficult to generalise the findings from Western studies in an Asian context (Joyce & Yates, 2007).

Vithal, Adler and Keitel (2005) point out that in South Africa, since the change of government in 1994, the most prominent phenomena in Mathematics education have been equity and social justice. Fundamental pedagogies as a philosophy no longer play such a big role. Education is expected to take into account the inadequacies of the past and learners need to be emancipated to become critical citizens. The inequalities in society and the transition to a democratic, non-racial, non-sexist, equitable society are emphasised. Teacher education is a focal point for changes in the education system,

and with the introduction of outcomes-based education (OBE), all teaching is required to focus on a general set of critical outcomes for education.

This study is concerned with the empowerment of South African learners through the teaching and learning of Mathematics. Ernest (2002, p. 1) describes three different but complementary domains of empowerment concerning Mathematics as ‘mathematical, social and epistemological’. A detailed description of this concept follows in section 3.5.3. Although the research in this study cannot ignore the phenomena of social justice and pedagogy in Mathematics education, the focus will be on the epistemological aspect of Mathematics education.

3.2.2 The nature of research frameworks

The Encarta World English Dictionary defines a framework as a set of ideas, principles, agreements, or rules that provides the basis or outline for something to be more fully developed at a later stage (McArthur, 1999). Lester (2005, p. 69) uses the metaphor of a scaffold as a framework:

A scaffold encloses the building and enables workers to reach otherwise inaccessible portions of it. Thus, a research framework is a basic structure of the ideas (i.e. abstractions and relationships) that serve as the basis for a phenomenon that is to be investigated. It helps to provide a structure for designing research studies, interpreting data resulting from those studies, and drawing conclusions.

Lester (2005) distinguishes between three kinds of frameworks: theoretical, practical and conceptual. Before identifying the framework of this study, each of these three kinds of frameworks is discussed briefly.

Within theoretical frameworks, the goal of the research and the data it gathers is to support, extend, or modify the theory in question, and the research questions are often rephrased in terms of this theory, and to some extent determined by it (Schoenfeld, 1998). One of the advantages of a theoretical framework is that it guides the research systematically and shows the progress in the research, but it can also have the disadvantage of sometimes forcing researchers to explain their results theoretically

rather than by evidence. This can lead to the misuse of data in the process of serving a theory, lacking relevance for everyday life, and not always providing the opportunity to integrate and validate the data (Arcavi, 2000; Lester, 2005).

A practical framework is based on “what works” (Lester, 2005, p. 71). The problems addressed are those of the people directly involved. A drawback is that the practical frameworks can at best only be generalised locally because of the narrow insider perspectives. These frameworks are to some extent the antithesis of theoretical frameworks (Lester, 2005).

Conceptual frameworks are based on both previous research and theory. Instead of relying on only one overarching theory, as theoretical frameworks do, they build on a variety of sources and can be based on different theories and various aspects of knowledge. The researcher decides what is relevant and important to address about a research question. Thus, in conceptual frameworks, it is the problems that drive the research and help identify the theoretical constructs that are used to build the scaffold (Lester, 2005).

Besides these three types of research frameworks, Jankvist (2011) mentions another possible type, namely data-driven research. Here the already accumulated data would be the point of departure. It is also possible to categorise research as method-driven, with the focus on the examination of a certain method. It can for example, be testing the particular method in a new setting or situation or in a different problem field (Schoenfeld, 1992).

At the start of this study it was far from a given which theory (or theories) should or could play a part in the research. The research question, “To what extent does the subject Advanced Programme Mathematics prepare learners for the rigour of first year Mathematics in the Science, Technology, Engineering, and Mathematics (STEM) university programmes?” was formulated before the researcher had any knowledge of the concept of a Mathematics education theory, and was rather based on the researcher’s teaching experience. Therefore, this research can be typified as problem-oriented and problem-driven research.

Cobb (2007, p. 29) argues for the advantages of using conceptual/investigative frameworks and suggests that “rather than adhering to one particular theoretical perspective, we must act as ‘bricoleurs’ by adapting ideas from a range of theoretical sources”. A conceptual framework would best guide the current study. The researcher considers herself a ‘bricoleur’, building a scaffold, a conceptual framework, that will help to make sense of and give meaning to the research process in which she engages.

3.2.3 Theories on mathematical learning

An attempt to review the literature on the theories of mathematical learning is complicated by lack of consensus on what constitutes a theory, as well as on what constitutes learning (Steffe, Nesher, Cobb, Sriraman & Greer, 2013). The Greek roots of the word ‘theory’ connect it to seeing or recognising, which explains why many authors claim that theories can be thought of as lenses (Cobb, 2007; Niss, 2007; Putnam, 1987). Theories show the set of assumptions underlying particular research, and determine the kinds of questions that are asked and the types of phenomena that are researched (Simon, 2009). Simon (2009, p. 481) describes the subtle difference between a theory and a worldview:

Worldview refers to the sum total of one’s beliefs, understandings, and deeply held commitments. It is only partially conscious and is related to one’s personal identity. Even though one’s worldview can evolve over time, the current state of one’s worldview has an impact on one’s actions, perceptions, thoughts, and emotions. Whereas one’s worldview is fairly well set at any particular point in time, I argue that theory use should be a matter of ongoing choice based on factors related to one’s research rather than one’s personal identity.

Schunk (1996) explains that “theories serve as bridges between research and educational practices and as tools to organize and translate research findings into recommendations for educational practice” (p. 27).

There is no one definition of learning that is universally accepted by theorists, researchers and practitioners (Shuell, 1986). Although people agree that learning is important, they hold different views on the causes, processes and consequences of

learning. Schunk (1996, p. 3) captures the criteria most educational professionals consider central to learning in the following definition of learning: “Learning is an enduring change in behaviour, or in the capacity to behave in a given fashion, which results from practice or other forms of experience”. This definition identifies three criteria of learning, i.e. that learning involves change, that learning endures over time and that learning occurs through experience.

Any theory on thinking, teaching or learning rests on an underlying philosophy of knowledge (Sriraman, 2002). The scientific study of learning had its beginnings in the writings of such early philosophers as Plato and Aristotle. Schunk (1996) describes the two prominent positions on the origin of knowledge as rationalism and empiricism. Rationalism can be traced to Plato, who distinguished knowledge acquired via the senses from that gained by reason. He argued that learning is recalling what exists in the mind, while information acquired with the senses constitutes raw materials rather than ideas. Empiricism refers to the idea that experience is the only source of knowledge. This position is derived from Aristotle (Plato’s student and successor) who believed that ideas do not exist independently of the external world, which is the source of all knowledge.

Mathematics education is situated between two fields of knowledge, i.e. Mathematics and Education, and both of these fields interact and are influenced by many other disciplines. This explains the complexity of developing theories that define mathematical learning (Sriraman & English, 2010). Much of the research in the field of Mathematics learning is founded on the learning theories of educational psychology (Ernest, 2010). Examples are Bloom’s taxonomy of educational objectives, Gagne’s behavioural objectives and learning hierarchies, Piaget’s stage theory, Ausabel’s advanced organisers and meaningful verbal learning - and later Vygotsky’s socially mediated learning and Simon’s artificial intelligence models for cognition. After the 1970’s a shift occurred beyond theory-borrowing towards theory-building, where theories draw on more than psychology. This is because the cohort of Mathematics researchers gradually became far more international and the research more multidisciplinary (Lesh, Sriraman & English, 2013).

Although an abundance of labels is used to describe the variety of learning theories, the major background learning theories can be categorised into two main areas, behavioural theories and cognitive theories. These theories differ in how they address the critical issues, such as how learning and transfer occurs, the role of memory and motivation, the processes involved in self-regulation, and the implications for instruction (Schunk, 1996).

Behaviourism was a powerful force in psychology in the first half of the twentieth century and most of the older theories of learning are behavioural. These theories view learning as a change in the rate, frequency of occurrence, or form of behaviour or response, which occurs primarily as a function of environmental factors (Schunk, 1996; Skinner 1953; Von Glasersfeld, 1995). The implications for educational practice are that behavioural theories are teacher- centred. This implies that teachers should arrange the environment so students can respond properly to stimuli. Although behaviourism does explain how behaviours change, it fails to account for how conceptual change occurs and does not explore mental processes or what is going on in human minds (Yilmaz, 2011).

In contrast, cognitive theories view learning as an active process of knowledge construction. They stress the acquisition of knowledge and skills, the formation of mental structures and the processing of information and belief (Schunk, 1996). Cognitive theories in educational practice underpin learner-centred instruction and take into account learners' perceptions of themselves and their learning environments. Teachers need to consider how instruction affects students' thinking during learning (Yilmaz, 2011). The works of Tolman, Piaget, Vygotsky, Bruner and German Gestalt psychologists were instrumental in the shift from behaviourism to cognitive theories (Lesh, Sriraman & English, 2013). Although cognitive theorists stress the importance of mental processes in learning, they disagree about which processes are important. Many researchers have shifted even more towards a focus on learners, and rather than talk about how knowledge is acquired, they talk about how knowledge is constructed. This theoretical perspective is referred to as constructivism (Schunk, 1996).

Constructivist theorists reject the idea that scientific truths exist and await discovery and verification. Knowledge is not imposed from outside people, but rather formed inside, within the setting of some environment (Clemons, 2006). In the classroom the teacher must structure teaching, learning experiences and the learning environments to challenge students' thinking so that they will be able to construct new knowledge (Ampadu & Adofo, 2014). Constructivist classrooms teach big concepts using much student activity, social interaction and authentic assessments. Students' ideas are avidly sought, and, compared with traditional classes, there is less emphasis on superficial learning and more emphasis on deeper understanding. Some instructional methods that fit well with constructivism are discovery learning, inquiry teaching, peer-assisted learning, discussions and debates, and reflective teaching (Schunk, 1996).

Constructivism has undoubtedly become a major theoretical influence in contemporary western educational reform efforts, especially in Mathematics, and takes on several forms: individual, social, cognitive and postmodern (Steffe & Gale, 1995). Ndlovu (2013, p. 10) describes constructivism as:

A more or less connected set of theoretic statements about the essential means by which mathematics learning can be described as a culmination of qualitative constructions and reconstructions based on prior understandings and resulting in new conceptual frameworks ... a good idea, insufficiently understood and badly implemented.

In South Africa the emphasis on constructivism principles in curriculum outcomes is a defining characteristic of OBE, especially in Mathematics (Ndlovu, 2013). The principles of constructivism therefore underpin the analyses that will be done to answer the first secondary research question of this study that aims to compare the different Mathematics curricula in South Africa.

In this study the discussion on the constructs that determine student performance will be based on the social cognitive theory of Bandura. Bandura and his colleagues studied observational learning, which came as another major challenge to behaviourism (Schunk, 1996). This theory stresses the idea that human learning occurs in a social environment and that people learn new actions merely by observing others. In

Bandura's theory a perspective of human behaviour is presented where persons can learn to set goals and self-regulate their cognitions, emotions, behaviours, and environments in ways that will enable them to achieve these goals. The main self-regulation processes that can occur prior to, during, and following task engagement are self-observation, self-judgment, and self-reaction (Bandura, 1977).

The final word on learning theories belongs to Biggs (1999, p. 5). He holds a constructivist approach and argues that meaning is created by the high school learner or student's approach to learning:

Learning is thus a way of interacting with the world. As we learn, our conceptions of phenomena change, and we see the world differently. The acquisition of information in itself does not bring about such a change, but the way we structure that information and think with it does. Thus, education is about conceptual change, not just the acquisition of information.

He distinguishes between a "deep approach (activities that are appropriate to handling the task so that an appropriate outcome is achieved), and a surface approach (activities of an inappropriately low cognitive level, which yield fragmented outcomes that do not convey the meaning of the encounter)" (Biggs, 2012, p. 3). As an example of a student with a deep approach he pictures someone who comes to class with relevant background knowledge and a question to be answered. The answer he finds forms the keystone for a particular branch of knowledge he is constructing. The student with a surface approach comes to class with no questions to ask, only wants to put in sufficient effort to pass, and sees the knowledge as one of the bricks to be recorded in his notes, which he must be able to remember in order to pass the examination. "Good teaching is getting most students to use the higher cognitive level processes that the more academic students use spontaneously. Good teaching narrows the gap" (Biggs, 1999, p. 4).

3.3 PERSPECTIVES ON CURRICULA

3.3.1 *The curriculum and student learning*

Biggs (1996) describes the ideal curriculum as one that is stated in the form of clear objectives stipulating the level of understanding required, as well as the content to be covered. The teaching methods chosen should be those likely to realise the objectives and the assessment tasks should determine whether the learning has been successful, and to convey to learners what the teacher wants them to learn. All these components support one another and address the same agenda, which means that the curriculum is well aligned. “The students are entrapped in this web of consistency, optimizing the likelihood that they will engage the appropriate learning activities” (Biggs, 1999, p. 11). In the alignment process verbs are used as markers to describe the perceived behaviour of students and they specify the levels of understanding that can be used for awarding levels of achievement (Biggs, 1999).

Scholars describe different manifestations of a curriculum, for example Goodlad (1979), who distinguishes between the ideal, the formal, the perceived, and the enacted, and English (2000), who distinguishes between written, taught and tested curriculums. There is consensus that these manifestations of the curriculum must all work together to ensure proper standards in an educational system (Trümpelmann & Nel, 1991). Squires (2008) points out that a well-aligned curriculum can help reduce the achievement gap between previously disadvantaged and advantaged learners.

In the tested curriculum, it is not always easy to distinguish between lower-order cognitive skills and higher-order cognitive skills. Farrell and Farmer (1980) observed that the crucial characteristic separating lower-order cognitive skills from higher-order cognitive skills is the element of novelty to the student or learner. Therefore, the teaching that had preceded the test influences the cognitive level of test questions. If a teacher used a level three question while teaching, and the students are exposed to a similar level three question in a test, then because of the familiarity, this would relegate the question to level two. This explains why only an insider can meaningfully assign particular questions to cognitive levels.

Hobden (2002) states that there is usually a narrow understanding of what it means to solve problems. He argues that the application of routine or normal strategies to find solutions, even to new questions, is not problem solving at all, since there is no reflecting, imagining, planning, categorising or reasoning involved. Even if students are unused to operating on higher cognitive levels, situations should be created where they can practise these skills in a non-test environment through practice and feedback (Felder & Brent, 2001).

3.3.2 The South African OBE Mathematics curriculum

(a) Foundation

The foundation of the NSC curriculum in South Africa is explicitly identified as outcomes-based education, which promotes a learner-centred, and activity based approach to education. It “serves to enable all learners to reach their maximum potential by setting the Learning Outcomes to be achieved by the end of the education process” (DoE, 2003 p. 2). The National Senior Certificate (NSC) in Mathematics focuses on learners who intend to continue with studies in Mathematics or who intend to enter into careers in which Mathematics is a requirement.

Parker (2006) describes the Mathematics curriculum as a “hybrid curriculum, one that exhibits features of a competence model as well as a performance model” (p. 12). On the general level as seen in the introduction and aims of the curriculum, the focus seems to be politically motivated and expresses the need for social justice for all and democratic access to Mathematics. However, the strong framing of the assessment standards and the contents indicate a need for explicit and visible criteria which are features of a performance-based pedagogy (Parker, 2006).

Parker (2006), in her analysis of the NCS, also found that the idea of empowerment as a purpose of Mathematics learning is visible in the curriculum document and that the focus seems to be on a structured form of applied Mathematics, including problem solving and mathematical modelling within different contexts and including real life. However, she argues that the idea of transferability of everyday knowledge into

Mathematics is absent. There is also an added focus on the historical aspects of the development and use of Mathematics in different cultures.

(b) Alignment

Any national school examination that is used for selection of learners for admission to further study or to measure a nation's mathematical competence needs a set of criteria that can be used to align the examination with the written curriculum and to determine its cognitive level (Berger, Bowie & Nyaumwe, 2010). The foundation of the new curriculum in South Africa is explicitly identified as OBE, which promotes a learner-centred and activity-based approach to education. A positive feature of this curriculum is that the Department of Education (DoE) published Subject Assessment Guidelines for Mathematics (SAGM) that outline the criteria and expected weightings of learning outcomes stated in the National Curriculum Statements (NCS) (DoE, 2008). Attention is therefore given to how things will unfold in practice; this is essential in a developing country such as South Africa (Rogan, 2007).

The DoE also published a SAGM taxonomy (based on the 1999 Trends in International Mathematics and Science Study [TIMSS] mathematics survey), designed for use in constructing and assessing the final examination (DoE, 2008). This taxonomy uses the categories of knowledge, routine procedures, complex procedures and problem solving. This is only one of many possible approaches when analysing the cognitive level of examination items, most of which are based on Bloom's taxonomy – the well-known hierarchy of six different levels of cognitive objectives (Bloom, 1956).

Berger, Bowie and Nyaumwe (2010) point out two difficulties in applying the SAGM taxonomy. They argue that the SAGM assumes that cognitive levels increase with the type of mathematical activity, which means that memorisation has the lowest cognitive level, then routine procedures, then complex procedures and then problem solving. This could lead to problems when assessing the cognitive levels of the examination items, and leaves no space for important mathematical activities such as justification, conjecturing, and communicating mathematical ideas, although these are important elements of the written curriculum. This could lead to a weak alignment of the examinations and the NCS. Secondly, the SAGM taxonomy cannot distinguish between

a complex procedure with high complexity and a complex procedure with moderate complexity.

(c) *The signalling ability of the NSC Mathematics curriculum*

The research of Engelbrecht, Harding and Phiri (2010) indicates a weak correlation between certain prior knowledge in areas required to pass university Mathematics and Grade 12 results. Jacobs (2010) emphasises that this could be because the students were unprepared for the level of cognitive skills required at university, or it could indicate that they did not put in sufficient effort. Volmink (2010), chair of UMALUSI, however, made it clear that the NCS serves as the end of a school phase and that the principles stated in the NCS make no mention of preparing learners for higher education.

Engelbrecht and others, (2010) determined that students coming from the OBE system had confidence in their abilities, were willing to try, were not prepared to follow the lecturer blindly, and wanted to experiment and do things their way. On the negative side, they also showed lack of mathematical rigour and had a unique way of writing unfamiliar to lecturers, and there was deterioration in their specific skills of factual knowledge, algebraic manipulation and mathematical formulation. It appears that the self-confidence they started out with in the courses was not justified because it was not supported by the necessary mathematical skills. There was a particular concern regarding the poor ability of students to 'write' Mathematics using the correct notation.

It seems that the students were underprepared for those topics that had been removed from the previous curriculum, such as absolute values and trigonometric functions. Although exponents and logarithms were taught in the school syllabus, the students' knowledge of these topics was insufficient and they could not keep up with the pace at university. The level of knowledge of functions was too low and the performance in the applications of differentiation was not promising for the follow-up topic of integration. It appears that in some university Mathematics courses the emphasis had shifted to a more theoretical approach, for which OBE students were not prepared. The overall conclusion of Engelbrecht and others (2010) was that the OBE curriculum widened rather than narrowed the gap between school and university Mathematics.

Kriek and Basson (2008), when reflecting on the content of the NCS, states that the inclusion of financial and statistics problem-solving skills enhances the curriculum and should create open-minded learners who are less method-bound than in the past. The fact that the learners are no longer required to learn formal proofs in either Algebra or Geometry has a negative impact on first-year Mathematics. The third paper (Probability and Geometry) requires real problem-solving skills and computational methodology, but not many schools write the paper or have staff who are trained to teach it (Jacobs, 2010).

The fact that the NSC Mathematics papers enforce lower time constraints (150 marks written in three hours, as opposed to the previous 200 marks in three hours) has an impact on the preparation of students for higher education (Jacobs, 2010). Green and Rollnick (2007) claim that time constraints are one of the reasons for testing at lower cognitive levels. Higher-order cognitive skills are unsuitable for limited time examinations because critical thinking takes time and testing in itself is a stressful event for students (Felder & Brent, 2001; Green & Rollnick, 2007).

In 2010, the UK's National Recognition Information Centre (UK NARIC) did a benchmarking analysis of the National Senior Certificate and comparisons with its international counterpart, the General Certificate of Education (GCE) A level. They acknowledged the social, political and cultural contexts that have influenced the NSC and still influence its evolution, and found that the NCS and NSC had developed extensively since 2005. Although the NCS Grades 10-12 had initially been criticised for a lack of subject specificity (Mhlolo, 2011) the development of the SAG have been universally welcomed by all stakeholders, especially teachers and examiners. The UK NARIC report found that the current problem of learners being unprepared in Mathematics cannot be laid at the door of the curriculum, but is rather a reflection of the uneven quality of the delivery of the curriculum. Evidence suggests that "candidates are increasingly more adept at taking initiative and displaying independent research and study skills. It is believed that this contributes to a gradual improvement in the all-round abilities of South African undergraduates" (UK NARIC, 2014, p. 106).

(d) Pedagogy

Although it is not within the scope of this study to discuss the acted curriculum in the South African classroom, the opinions of two scholars, Parker and Ndlovu can be mentioned. Parker finds that the words “learner-centred” and “activity –based” are used exactly once in the NCS Mathematics document (in the rationale) and that the meanings thereof are never explained. This has led to many different interpretations of the concepts by teachers in the classroom (Parker, 2006).

Ndlovu (2013) writes that it appears that the teaching and learning methods used in OBE Mathematics classrooms are increasingly being questioned for their effectiveness, and therefore there is a need for rethinking the efficacy of the underpinning constructivist learning theory.

3.3.3 Thinking skills required in the transition from school to university Mathematics

(a) The transition period

To discuss the full range of thinking skills required for success in Mathematics at university level is a major task that cannot be accomplished in this study. However, it is necessary to look at the basic skills required in the transition from school to university. Gueudet (2008) defines the period of two years before entering university and two years after entering as the period when transition issues become evident. He notes that Mathematicians and Mathematics educators often mention transition from school to university as a major issue. Authors researching transition issues over time, distinguish between epistemological, socio-cultural and didactical and cognitive perspectives, but student difficulties always form the central argument.

Tall (1991) declares that the word ‘transition’ is extensively used in literature belonging to the Advanced Mathematical Thinking (AMT) field. In the last decade, many scholars have debated if the term ‘advanced’ refers to the Mathematics, the thinking, or both. Many authors agree that advanced thinking can occur at any level, but that there is a specific need for advanced thinking when university Mathematics is attempted (Gueudet, 2008).

(b) The secondary-tertiary transition in Mathematics

Tall (1991, p. 20) defines the movement from elementary Mathematics to AMT as involving a significant transition: “from describing to defining, from convincing to proving in a logical manner based on definitions ... when a student begins to deal with abstract concepts and deductive proof. Robert (as cited in Gueunet, 2008) argues that the mathematical content taught at the end of high school and the beginning of university is intrinsically difficult because it involves formalising, unifying, generalising and simplifying.

Clark and Lovric (2009) describe a successful transition as one where a student feels comfortable in his/her new role as university student, is able to work and achieve according to his/her goals, shows good academic progress, has support (academically and otherwise) and can access it when needed and enjoys the Mathematics courses. A more detailed discussion of notion ‘student success’ will follow in section 3.4.

(c) Thinking skills required for success in university Mathematics

The general decline in undergraduate mathematical ability has been the subject of many reports since the mid-1990's. A 1995 report of the London Mathematical Society states that

There is unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates. The serious problems perceived by those in higher education are: (a) a serious lack of essential technical facility – the ability to undertake numerical and algebraic calculation with fluency and accuracy; (b) a marked decline in analytical powers when faced with simple problems requiring more than one step; (c) a changed perception of what Mathematics is – in particular of the essential place within it of precision and proof (Howson et al., 1995, p. 20).

More recently, Clark and Lovric (2009) mention that because of the changes in technology, university students should be able to produce much more productive thinking instead of just reproducing algorithms in appropriate circumstances, which technology can now address. Other cognitive difficulties that Mathematics students

experience are formal proof and working with theorems, difficulties related to concepts such as limits, infinity and functions, and the development of strategies for reasoning, such as generalizing, property abstraction or working from definition. The hardest obstacle is however adapting to a new learning strategy, which is the transition from surface learning to deep learning (Clark & Lovric, 2009).

Many new university students rely on imitative reasoning when doing mathematical tasks. This means copying solutions to tasks, for example by remembering an algorithm or an answer or looking at a textbook example (Bergqvist, 2007). This type of reasoning too often weakens their understanding of underlying mathematical concepts and actually prevents them from learning more powerful mathematical concepts, like problem solving and deductive reasoning. In some cases it is possible for students to solve up to 70% of calculus textbook exercises using imitative reasoning and many students can pass most first year Mathematics exams using only imitative reasoning. As assessment in general affects how students study, the design of the exams influence the student's skills and attitudes. Students should therefore be given more opportunities to learn creative reasoning and become familiar with a situation with unfamiliar tasks (Bergqvist, 2007).

Moore (1994) observes seven kinds of difficulties first-year students experience with proof: they do not know the definitions or how to apply them, they have little understanding of the concepts, they are unwilling or unable to generate their own examples, they do not understand mathematical language or notation and they do not know how to start the proofs.

Since the discipline of engineering consists of problem-solving approaches and problem-solving thinking skills, it is important for a student in engineering to possess a conception of Mathematics as problem-solving rather than simply a list of algorithms (Craig, 2013). An important finding is that of Attridge and Inglis (2013) that the post – compulsory pre-university study of Mathematics develops conditional reasoning skills and that taking an advanced Mathematics course is associated with the development of logical reasoning.

3.4 STUDENT SUCCESS AT UNIVERSITY

In order to answer the question on the effectiveness of Advanced Placement (AP) in preparing students for success in first-year university Mathematics, it is necessary to define first what is meant by the notion ‘success’ before giving an overview of the most prominent theories on student success.

3.4.1 *The concept ‘student success’*

Defining student success is not an easy task. Throughout the literature of the past 100 years it has been closely interwoven with concepts such as student persistence, student attrition and student retention, and these are some of the most widely studied concepts in higher education (Bean, 1980; Kuh, 2003; Seidman, 2005; Tinto, 1987). Mostly retention is used as an institutional measure and persistence as a student measure. In other words, institutions retain and students persist. Attrition is defined as the diminution in numbers of students resulting from lower student retention (Seidman, 2005). Student persistence refers to the enrolment of an individual over time that may or may not be continuous and may or may not result in degree completion, while student success is a broader concept that places a value on the different forms of persistence – the most common being to complete a degree (Tinto & Pusser, 2006).

When defining student success, it should always be kept in mind that students enter a university from different social, cultural and academic backgrounds. Tinto and Pusser (2006) argue that a student enters a university with a variety of attributes (gender, social class, race, and ethnicity), abilities, skills, levels of prior academic preparation, attitudes, values and knowledge about higher education (e.g. goals, commitments and expectations). There is also a range of external notions (e.g. family, work, community and other students), which each has an influence on a student’s energy and time. The institution they study at has its own unique attributes such as level, size, mode of control and resources. Together all these factors influence student learning and the quality of student effort, which again shape student success in the classroom.

3.4.2 *Prominent theories on student success*

Tinto's first student integration model, published in 1975, is seen as a landmark in research on student attrition. This model theorises that students who integrate socially into a campus community increase their commitment to the institution and are more likely to graduate (Tinto, 1975). Over the course of 35 years Tinto continued to publish and upgrade his models. More recent versions have included motivational variables including goal commitment, the need to match student expectations to institutional mission, the transition of students through the university process and retention models for multicultural and minority student groups (Demetriou & Schmitz-Sciborski, 2011).

Throughout the 1980's notable theorists such as Bean (1980) and Astin (1984), stressed the importance of background characteristics such as prior academic performance, distance from home and socio-economic status as factors influencing student retention. Astin's model on student retention involves student demographics and prior experiences, student experiences at university and individual student attributes, including knowledge, attitudes and beliefs (Pascarella & Terenzini, 2005).

Much of the retention literature of the 1990's focuses on encouraging retention for students of colour, underrepresented populations and previously disadvantaged students (Swail, 2004). Tinto (1999) argued that academic counselling is imperative to undergraduate retention because it keeps students motivated, stimulated and working towards meaningful goals (Demetriou & Schmitz-Sciborski, 2011).

Over the last decade many studies have explored new models of retention in a more holistic way, including students' psychologically motivated behaviours, a socially constructed context in which to make meaning of academic integration. Researchers looked at persistence as engaging all aspects of a student's life. Engaged students develop habits of the mind and heart that engage their capacity for continual learning and personal development (Kuh, 2003). Motivational theories from multiple fields of study, including educational psychology and social psychology, have been used to gain understanding of student persistence and retention (Demetriou & Schmitz-Sciborski, 2011). These theories, for example, include the attribution theory of motivation (Zimmerman & Schunk, 2006), the expectancy theory (Vroom, 1964), the expectancy–

value theorem (Eccles & Wigfield, 2002), the goal-setting theory (Locke & Latham, 2006), self-efficacy beliefs (Bandura, 1977), the academic self-concept theory (Schavelson & Bolus, 1982), the motivational orientation theory (Baker, 2004) and theories about optimism (Peterson, 2000).

When discussing student success, it is also necessary to look at the work of two recent South African philosophers, Morrow and Hlalele. In 2002 Morrow coined the term “epistemological access”, which signals meaningful access to the ‘goods’ of the university (Muller, 2014). “To learn to become a participant in an academic practice is to learn the intrinsic disciplines and constitutive standards of the practice” (Morrow, 2009, p. 77). Epistemological access is learning how to become a successful participant in an academic practice. “In the same way in which no one else can do my running for me, no one else can do my learning for me” (Morrow, 2009, p. 78).

Hlalele (2010) points out that variables such as academic self-concept, academic self-efficacy, and academic locus of control may contribute immensely to academic success. He cautions that enrolment at a university does not necessarily guarantee success in subsequent degree programmes and that universities should endeavour to harmonise access with acceptable throughput levels and make an effort to develop epistemological access to students.

From the discussion above it is obvious that the domain of student performance and persistence is widespread and multidimensional. It is therefore necessary to make a few assumptions to delimit the scope of this study and to define student success for this controlled situation.

This study is done at only one university, i.e. Stellenbosch University, and the focus is on the Mathematics courses that are pre-requisites for majors in the STEM sciences. It is assumed that students who took the APM course in high school are academically gifted students who had access to a school that teaches APM or who had the financial resources to enrol for an online AP course. For the purpose of this study, academic success is defined quantitatively and seen as the ability of a student firstly to persist throughout his first year, and then subsequently to obtain the minimum requirements to pass the first-year course in Mathematics and eventually to graduate.

3.4.3 Variables predicting students' academic success

When discussing variables that predict academic success and persistence, it is necessary to identify the cognitive or traditional academic preparedness variables together with non-cognitive variables, both of these in combination with the bio-and socio-demographic variables (Ackerman, Kanfer & Beier, 2013). Within the scope of this study, it will not be possible to discuss the bio- and socio-demographic variables; the focus will be on theories of cognitive and non-cognitive variables that predict success in general, specifically in Mathematics.

3.4.4 Cognitive ability as a determinant of academic achievement

There are various representations of the concept 'general mental or cognitive ability'. Jacobs (2010) distinguishes between ability (a personal power, skill, mental capacity or cleverness), aptitude (indicates future achievement), intelligence (indicates learning ability), mental ability (indicates mental processes required for learning) and proficiency or achievement (measured against what has been learned, thus prior knowledge).

Understanding the reasons for individual differences in levels of scholastic achievement has always been a concern of educational psychologists. A large body of research has established measures of cognitive ability as important predictors of academic success (e.g. Ackerman & Heggestad, 1997; Carpenter, Just & Shell, 1990; Gardner, 1993). Already in 1916, Binet and Simon used the psychology of individual cognitive capacities to explain variability in children's academic performance. This gave rise to extensive research into intelligence and intelligence testing (Neisser et al., 1996), and the development of a wide range of assessment instruments with which to identify these individual differences. Predicting performance depends on being able to assess it.

Traditionally the Grade 12 average and standardised test scores, such as Standard Assessment Tests (SAT) (used in the USA), A Level marks (used in the UK) or the South African National Benchmark Tests (NBT) are the strongest predictors of university performance and also the instruments that most universities use as part of their admission criteria (Astin, 1993). Although intelligence tests have achieved wide acceptance as tools to predict future success, many researchers have challenged this

notion and presented alternative theories. Standardised intelligence test scores and high school grade point average (GPA) have been widely criticised as barriers to the enrolment of non-traditional students and insufficient to explain the persistence and success of all students (Jaeger, Bresciani, & Sabourin, 2002).

Because student selection, especially at selective institutions, reduces variation in intelligence scores (Furnham, Chamorro-Premuzic & McDougall, 2002), non-intellective or non-cognitive factors may be critical to accurate prediction of performance (Richardson, Abraham & Bond, 2012). Despite the long history of research in the field of improving academic performance and retention, attributes other than intellectual ability are merely suggested as potential influences and only a few robust psychological instruments have been developed for the prediction of academic success at tertiary level (Ackerman et al., 2013). Recent meta-analyses that made a valuable contribution in exploring the role of individual attributes and study skills in predicting academic performance are those of Poropat (2009), Richardson et al., (2012) and Robbins et al., (2004).

3.4.5 *Non-cognitive predictors of academic achievement*

Over the past 20 years many large-scale international studies in education, like TIMSS and the Programme for International Student Assessment have focussed on assessing different aspects of academic performance. Stankov (2014) notes that the background questionnaires of these studies contain a wealth of information on non-cognitive variables of potential importance to education. Some of the largest effects on educational achievement are due to these psychological (i.e. in-persons) non-cognitive individual traits (Ackerman, Bowen, Beier & Kanfer, 2001).

Allport (1927) defines a trait as habitual patterns of behaviour, thought and emotion. It can also be referred to as a relatively stable characteristic or construct and is an independent statistical indication that can be self-assessed or assessed by peers (Stankov, Morany & Lee, 2014). Whereas cognitive ability reflects what a person *can* do, traits reflect what a student *will* do (Furnham & Chamorro-Premuzic, 2004).

Of particular interest for this study is the extensive research into the correlates of tertiary-level academic performance by Richardson and others (2012) because it is necessary to be able to determine the influence of an intervention such as APM on an individual's traits. Richardson and others (2012) attempted to identify which individual differences are associated with better performance, how strong these associations are and whether a dependable model of predictors can be constructed on such findings. They based their findings on studies into psychological correlates of university students' academic performance published between 1997 and 2010. Their systematic search of a database consisting of 7167 English-language articles yielded 50 conceptually distinct correlates, which included three demographic factors, five traditional measures of intelligence and prior academic performance and 42 non-intellective constructs.

Very useful was their classification of the non-intellective constructs into five basic conceptually overlapping domains: a) personality traits, b) motivational factors, c) self-regulatory learning strategies, d) students' approaches to learning, and e) psychosocial contextual influences. Personality traits include the so-called Big Five i.e. neuroticism, extraversion, openness to experience, agreeableness and conscientiousness, while motivational factors include constructs such as locus of control, academic self-efficacy, performance efficacy and grade goal. Students' approaches to learning include deep, surface, and strategic approaches, while self-regulatory learning includes critical thinking, meta-cognition and test anxiety.

A meta-analysis of 50 constructs reveals that 41 of them were significantly associated with university students' grade point average (GPA). The demographic and psychosocial contextual factors generate only small correlations with students' academic performance, while past scholastic achievements produce medium-sized correlations. The research concludes that a very strong predictor of tertiary performance is a combination of motivation variables, i.e. self-efficacy, performance efficacy, grade goal and self-regulatory capacity (locus of control). Together these constructs account for 14% of the variance in university grades. The strongest individual correlate observed is for performance self-efficacy. This is consistent with the findings of Pajares and Miller (1995) and Lee and Stankov (2012).

Another research approach where the individual traits are integrated (integrative trait-complex approach) and used in conjunction with cognitive, affective and conative traits of success, emerged over the past 50 years (Ackerman, Chamorro-Premuzic & Furnham, 2011). The term 'aptitude complexes' was articulated by Snow (1987) and meant a combination of levels of some variables that facilitate either efficient learning, do not facilitate, or might even impede efficient learning. In general, empirical studies have shown that the underlying synergy of the integrated traits together are better predictors of academic performance variables, in comparison to approaches that consider individual trait measures in isolation (Ackerman et al., 2011).

The three domain-specific self-beliefs – self-efficacy, self-concept and anxiety-, and the predictive power of these self-beliefs on students' academic achievement in specifically Mathematics have been developed and studied extensively by educational psychologists (Lee, 2009; Pajares & Schunk, 2002; Stankov, 2014). Students' self-efficacy in Mathematics refers to the extent to which the students believe they can do the task (Bandura, 1994). Students' anxiety in Mathematics refers to the extent to which students feel helpless or stressed and students' self-concept in Mathematics refers to the extent to which they feel competent in the domain being tested (Meece, Wigfield & Eccles, 1990). Of these three self-beliefs, the best predictor in relation to Mathematics achievement is self-efficacy (Lee & Stankov, 2012).

3.5 THEORIES ON STUDENT SUCCESS IN MATHEMATICS

The broad theoretical base underpinning this study's section on student success in Mathematics consists of the social cognitive theory of the psychologist Bandura (1977) and the writings of Ernest (2002) on mathematical empowering of learners. Bandura's theory is elaborated on by the recent research of the Australian psychologists, Stanlov and Lee, on self-efficacy and self-confidence. The findings of the American psychologists, Ackerman, Kanfer and Beier (2013), give important direction on the role of AP programmes in predicting student success. The next section will discuss these four theories.

3.5.1 Bandura's self-efficacy theory

(a) The social cognitive theory

Theories and empirical studies on the three self-beliefs in the domain of mathematics, self-efficacy, anxiety and self-concept, are all grounded in the social cognitive theory of Bandura (1977). This theory states that individuals possess a self-system that enables them to exercise a measure of control over their thoughts, feelings, motivation, and actions. This self-system serves a self-regulatory function by providing individuals with the capability to influence their own cognitive processes and actions and thus alter their environments (Bandura, 1986).

Individuals create and use these self-beliefs intuitively. They engage in behaviour, interpret the results of their actions, use these interpretations to create and develop beliefs about their capability to engage in subsequent behaviours in similar domains, and behave in accordance with the beliefs created. In school, for example, the beliefs that students develop about their academic capabilities help determine what they do with the knowledge and skills they have learned. Consequently, their academic performance is in part the result of what they come to believe they have accomplished and can accomplish. This helps explain why students' academic performance may differ markedly when they have similar abilities (Bandura, 1986).

(b) The concept 'self-efficacy'

The self-efficacy extension of Bandura's social learning theory was first introduced in the seminal publication, *Self-efficacy: Toward a Unifying Theory of Behavioural Change*. A decade later, Bandura (1986) situated the construct within a social cognitive theory of human behaviour. In 1997 Bandura published a book, *Self-efficacy: The Exercise of Control*, in which he discussed the nature and structure of self-efficacy beliefs, their origins and effects, the processes through which self-beliefs operate and the modes by which they can be created and strengthened. Over two decades, this theory of self-efficacy was widely tested in various disciplines and settings, but specifically in educational research studies on academic motivation and self-regulation (Pintrich & Schunk, 1996).

Bandura (1977) defines self-efficacy expectations as a person's beliefs concerning his/her abilities to perform a given task or behaviour successfully. Self-efficacy expectations are a major determinant of whether a person will attempt a given task, how much effort will be extended, and how much persistence will be displayed in pursuing the task in the face of obstacles. Being sure of oneself facilitates success in what one attempts and accomplishes in the future (Bandura, 1999). This does not mean that people can accomplish tasks beyond their capabilities simply by believing that they can, for competent functioning requires harmony between self-beliefs on the one hand and skills and knowledge a person possesses on the other. It means that self-perceptions of capability help determine what individuals do with the knowledge and skills they have. Self-efficacy beliefs are critical determinants of how well knowledge and skill were acquired in the first place (Pajares, 1997).

Self-efficacy judgments are more task- and situation-specific and are used in reference to some goal. This distinguishes them from related conceptions of personal competence that form the core constructs of other theories. To understand the nature of self-efficacy beliefs better, it may be useful to explain how they are acquired.

(c) Sources of self-efficacy

Perceptions of self-efficacy are acquired from four sources of information, of which the first and most influential are an individual's personal accomplishments, also seen as mastery experiences. Individuals measure the effects of their actions, and their interpretations of these effects help create their efficacy beliefs. Outcomes interpreted as successful raise self-efficacy; those interpreted as failures lower it. The second is verbal persuasion that an individual receives from others. Persuaders must cultivate people's beliefs in their capabilities while at the same time ensuring that the envisioned success is attainable. Just as positive persuasions may work to encourage and empower, negative persuasions can work to defeat and weaken self-beliefs. In fact, it is usually easier to weaken self-efficacy beliefs through negative appraisals than to strengthen such beliefs through positive encouragement (Bandura, 1986).

Thirdly, self-efficacy information is the vicarious experience of the effects produced by the actions of others. It is also called observational learning where a significant model in

one's life can help instil self-beliefs that will influence the course and direction that life will take. An individual's vicarious experience also involves the social comparisons made with other individuals and peers. The fourth source of information on self-efficacy beliefs are physiological states such as anxiety, stress, arousal, fatigue and mood. Strong emotional reactions to a task provide clues about the anticipated success or failure of the outcome (Bandura, 1977; Bandura, 1986).

(d) Interventions to increase self-efficacy

Interventions to increase student achievement should therefore focus on altering students' beliefs of their self-worth or competence. This is usually accomplished through programmes that emphasise enhancing self-beliefs through verbal persuasion methods. Students who believe they have the skills and abilities to succeed at academic tasks perform better than students with lower efficacy expectations (Bandura, 1997). Efficacy expectations for any particular performance depend on a students' experience with similar challenges. When challenges are familiar, students can draw upon past experience to formulate expectations about specific performance.

(e) The relationship between self-efficacy beliefs and academic performance

Research findings over the past 20 years have generally supported Bandura's theory that efficacy beliefs mediate the effect of skills or other self-beliefs on subsequent performance attainments (Bandura, 1997; Schunk, 1991). Bong, Cho, Ahn and Kim (2012) argue that students with a strong sense of self-efficacy tend to involve themselves in challenging tasks, invest more effort and persistence, and show excellent academic performance in comparison to students who lack such confidence. Students with strong self-efficacy are less likely to abandon difficult tasks than those who have doubts about their abilities (Ackerman, 2005). In studies of university students who pursue science and engineering courses, high self-efficacy has been demonstrated to influence the academic persistence necessary to maintain high academic achievement (Lent, Brown, & Larkin, 1986).

(f) *The relationship between self-efficacy beliefs and other motivation constructs*

Several researchers have shown that self-efficacy beliefs influence academic performance by influencing effort, persistence and perseverance (Bandura & Schunk, 1981; Bouffard-Bouchard, 1990; Schunk & Hanson, 1985). Bouffard-Bouchard, Parent and Lavirée (1991) found that students with high self-efficacy engaged in more effective self-regulatory strategies at each level of ability. Self-efficacy also enhances students' memory performance by enhancing persistence (Berry, 1987). Academic self-efficacy influenced achievement directly ($= .21$) as well as indirectly by raising students' grade goals ($= .36$) (Wood & Locke, 1987). Pintrich and De Groot (1990) reports a correlation between academic self-efficacy and both cognitive strategy use and self-regulation through use of metacognitive strategies. Students who believe they are capable of performing academic tasks use more cognitive and metacognitive strategies and persist longer than those who do not (Pintrich & Garcia, 1994).

(g) *Mathematics self-efficacy*

Mathematics self-efficacy can be distinguished from other measures of attitudes towards Mathematics in that Mathematics self-efficacy is a situational or problem-specific assessment of an individual's confidence in his/her ability to perform a particular task successfully (Hackett & Betz, 1985). Schunk (1984) reports that Mathematics self-efficacy influenced Mathematics performance both directly ($= .46$) and indirectly through persistence ($= .30$).

Self-efficacy perceptions mediate between prior attainments and academic performance, which means that students with similar prior achievements and cognitive skills may differ in subsequent performance as a result of differing self-efficacy perceptions. In consequence, such performance is generally better predicted by self-efficacy than by prior academic achievements (Schunk, 1991). Pajares and Miller (1995) report that mathematics self-efficacy has stronger direct effects on problem solving than do self-concept, perceived usefulness or prior experience. Perceived positive family encounters also directly predict creative problem-solving practices in Mathematics as they enhance confidence in intelligence and intrinsic motivation (Cho &

Lin, 2010). Both Bandura (1986) and Pajares and Miller (1995) argue that students' judgments to solve Mathematics problems should be more strongly predictive of their capability to solve those problems than should their confidence to perform other Mathematics-related tasks or their confidence of earning A's or B's in Mathematics-related courses.

Bandura (1986) argues that the stronger the self-efficacy, the more likely persons are to select challenging tasks, persist at them, and perform them successfully. Efforts to lower students' efficacy perceptions or interventions designed to raise already overconfident beliefs should be discouraged, but improving students' 'calibration' - the accuracy of their self-perceptions - will require helping them to understand better what they know and do not know, so that they may more effectively deploy appropriate cognitive strategies as they perform a task. Conversely, students who lack confidence in skills they possess are less likely to engage in tasks in which those skills are required, and they may give up easily in the face of difficulty.

(h) The causal predominance of self-efficacy

In self-concept research, the issue has been one of whether feeling good about oneself is primarily responsible for increased achievement or whether successful performance is largely responsible for stronger feelings of self-worth. Because of the reciprocal nature of human motivation and behaviour, it is unlikely that such a question can be resolved (Bandura, 1986; Eccles & Wigfield, 1985). Findings from investigations in which this has been examined suggest that self-efficacy beliefs make a causal contribution to the level and quality of human functioning (Bandura, 1997), and these beliefs can be altered using vicarious methods, verbal persuasion, differing performance feedback or social comparative information, and/or by manipulating task complexity.

(i) Collective efficacy

Bandura (1986) provides valuable insight when he observes that confidence is both a personal and a social construct, and that collective systems such as classrooms, teams of teachers, schools, and school districts develop a sense of collective efficacy - a group's shared belief in its capability to attain its goals and accomplish desired tasks.

Students, teachers and school administrators operate collectively as well as individually. As a result, schools develop collective beliefs about the capability of their students to learn, of their teachers to teach and otherwise enhance the lives of their students, and of their administrators and policymakers to create environments conducive to those tasks.

Erikson (1980, p. 95) put it this way:

Children cannot be fooled by empty praise and condescending encouragement. They may have to accept artificial bolstering of their self-esteem in lieu of something better, but what I call their accruing ego identity gains real strength only from wholehearted and consistent recognition of real accomplishment, that is, achievement that has meaning in their culture.

In South African schools the collective beliefs about education are influenced strongly by the political beliefs on education and the concern for power. In many disadvantaged schools the political culture of schools is the result of a “deadly collision between the apartheid legacy of disgust and the post-apartheid inheritance of distrust” (Jansen, 2013, p. 93).

(j) Self- efficacy beliefs and the mathematically gifted student

Pajares (1996) used path analysis to test the role of self-efficacy beliefs in mathematical problem solving of gifted children. He controlled for the effects of general mental ability, Mathematics concept, Mathematics anxiety, previous performance in Mathematics and sex. The key finding was that the influence of these determinants on academic performance diminishes when particularised assessments of self-efficacy are included in a model. Other results were that gifted girls performed better than gifted boys in performance but did not differ in self-efficacy. The gifted students showed higher Mathematics self-efficacy and self-efficacy for self-regulated learning, as well as lower Mathematics anxiety than did regular education students. Although most students were overconfident about their capabilities, gifted students had more accurate self-perceptions and gifted girls were biased toward under-confidence. Results support the hypothesised role of self-efficacy in Bandura's (1986) social cognitive theory.

3.5.2 The self-confidence theory of Stankov

(a) Self-confidence

As shown in 3.5.1, until very recently most efforts to predict academic achievement in the mathematical domain focused on self-beliefs and specifically on self-efficacy. In 2013 a relevant theoretical study done by Stankov, Morany and Lee (2014) identified a new area that lies between the study of cognition and personality, i.e. the area of self-confidence. They argue that confidence can be classified as a fourth self-belief together with self-efficacy, anxiety and self-concept, and that it has evolved through the interaction between the social and the physical environment.

The next question that can be asked is which self-belief is then the best non-cognitive predictor of success, confidence or self-efficacy, and what the difference between the two self-beliefs is. The answer to this question will underpin the argument in this study on the effectiveness of the subject APM in furthering success in first-year Mathematics. It will help teachers of APM to focus on constructs that increase student achievement.

Although measures of confidence and measures of self-efficacy are conceptually related, they remain different constructs. Confidence measures are more domain-specific than self-efficacy measures. In measures of confidence one evaluates the success of the immediately preceding act, whereas in self-belief measures there is a need to compare oneself with some similar acts carried out in the past, with other people or with one's own performance on different tasks. The main difference between the two measures is whether the item has been attempted (Stankov, 2014).

(b) Stankov and Lee's gradient of predictability

Stankov and Lee (2014) put together the information from new studies with relatively large samples of participants on constructs that were relevant and important for student outcomes. They proposed a model according to which the predictive validity of non-cognitive measures in relation to achievement and cognitive ability can be arranged from poor to strong. The four steps in the gradient of predictability from bottom to top are:

- *Psychological constructs independent of cognitive performance*
Broad measures of maladjustment, i.e. general depression, well-being, and toughness and motivation/goal orientation, which have low or negative correlations with achievement.
- *Psychological constructs that reflect moderate engagement in cognitive activities*
Measures such as openness to experience and self-concept measured in a specific domain such as Mathematics have moderate predictive validity for academic achievement in Mathematics.
- *Psychological constructs that reflect strong engagement in cognitive activities*
Measures such as self-efficacy that correlate strongly with mathematics achievement and mathematics anxiety have a moderate/high correlation with academic success.
- *Psychological constructs tapped by judgements of the quality (confidence) of one's recent cognitive history.*
This gradient of predictability shows that the best predictors of any kind of cognitive performance are measures of confidence, "a self-evaluative belief in the correctness of one's cognitive act" (Stankov & Lee, 2014, p. 3).

The research determined a raw correlation of .68 between confidence and Mathematics, and this concludes that when confidence is treated as a self-belief construct, it is the best non-cognitive predictor of success. Morony, Kleitman, Lee and Stankov (2013) argue that it is possible for a confidence measure to capture a major part of predictive validity of the other self-beliefs. This also confirms studies by Lee (2009) and Stankov, Lee, Luo and Hogan (2012). Stankov, Morany and Lee (2014, p. 12) claim that:

Confidence on its own can explain more variance in Mathematics performance than all other self-belief measures combined – and the lack of substantial incremental variance accounted for by the other self-constructs suggest[s] that a single confidence measure can be used when the objective is to predict achievement in Mathematics.

Since confidence is measured in connection with a just completed cognitive act (Stankov, 2014), it can be a useful intervention to provide feedback on the accuracy of a

test item. It might change the level of confidence and be particularly useful for students at lower levels of ability who are known to show over-confidence. Another approach may involve emphasis on reflective thinking and training in the use of metacognitive skills of planning, monitoring and evaluation of one's cognitive performance. Such training may help students to adjust their own confidence and self-beliefs (Stankov, 2014). In this sense confidence can also be classified as a metacognitive trait. An aim of the current study will be to determine if APM course-taking develops confidence in students' ability to do Mathematics and helps them to be able to reflect on their own performance in Mathematics.

These recent findings on confidence support the findings of an American philosopher, Ernest, who states that having confidence leads to epistemological empowerment with concern to Mathematics. To clarify this concept, the next section will discuss his theory on mathematical empowerment.

3.5.3 *The mathematical empowerment theory of Ernest*

Empowerment is "the gaining of power in particular domains of activity by individuals or groups and the processes of giving power to them, or processes that foster and facilitate their taking of power" (Ernest, 2002, p. 1). This study seeks to determine the extent to which the subject APM can empower a learner in his/her learning of Mathematics.

Ernest (2002) distinguishes three different but complementary domains of empowerment concerning Mathematics and its uses, i.e. mathematical, social and epistemological empowerment.

(a) *Mathematical empowerment*

A person is mathematically empowered if he/she has power over the knowledge and skills, as well as over the language and symbols of Mathematics and can confidently apply them in school Mathematics. He/she will be able to demonstrate a wide range of cognitive capabilities such as performing algorithms and procedures or applying mathematical strategies.

Mastery over the knowledge and skills of Mathematics is the more traditional psychology perspective, where a successfully empowered learner has certain cognitive mathematical capabilities such as applying and using general facts, skills, concepts and all forms of mathematical knowledge, application of strategies and carrying out plans to solve problems (Ernest, 2002). This also involves meta-cognition, the management of one's own cognitive processes (Ernest, 2002; Flavell, 1976; McMillan, 2001). Cognitive strategies are used when a Mathematics problem is solved, but metacognitive processes are employed when learners are aware of their thinking about the problem, or when they begin to evaluate their progress in solving the problem. In practice, learners constantly alternate between metacognitive and cognitive processes (Larkin, 2009).

Mastery over the language and symbols of Mathematics is the semiotic perspective where the successfully empowered learner has the ability to “make sense of, write and judge the correctness of mathematical texts concerning mathematical tasks and questions as well as their solutions and answers, including asking the questions themselves” (Ernest, 2002, p. 3).

(b) Social Empowerment

“Social empowerment through Mathematics concerns the ability to use Mathematics to better one's life chances in study and work and to participate more fully in society through critical mathematical citizenship” (Ernest, 2002, p. 4). Examination and test results or certificates in Mathematics often open doors of opportunities to advanced studies and several rewarding occupations not only in STEM courses, but also in other highly paid occupations, such as the caring professions, financial services and management positions (Ernest, 2002).

Many researchers have noted the role of Mathematics as a ‘critical filter’ controlling entry to higher education and higher paid occupations (Ma & Johnston, 2008; Stinson, 2004). In South Africa, there has been a lot of debate on the ‘gatekeeper’ role of Mathematics. It is argued that Mathematics has disempowered and excluded many previously disadvantaged learners from higher education and its privileges, because it did not provide them with the “key to the gate” (Stinson, 2004, p. 4). Critics argue that

the post-democracy goals of transformation and equity have not been achieved in practice (Fleisch, 2008; Jansen & Sayed, 2001; Volmink, 1994). Skovsmose (2000) in his writings raises awareness of the two faces of Mathematics – on the one hand granting inclusion and empowerment, and on the other hand leading to oppression, exclusion and disempowerment.

Social empowerment also concerns critical mathematical citizenship, where students view the world critically, can use their mathematical knowledge and skills to think independently, see the detailed as well as the bigger picture, and make balanced judgments. Hopefully this will “lead to the promotion of social justice and a better world for all” (Ernest, 2002, p. 6).

(c) *Epistemological empowerment*

Epistemological empowerment concerns the individual’s growth of confidence not only in using mathematics, but also a personal sense of power over the creation and validation of knowledge. For a learner to obtain epistemological empowerment through mathematics, it must over a long term become an integral part of his personal identity (Ernest, 2002).

Ernest’s description of what it means when Mathematics becomes an integral part of a learners identity is critical to the argument in this study. He argues that learners need to:

- *be confident in their mathematical knowledge and skills;*
- *be confident in their ability to apply these capabilities both in routine and non-routine mathematics tasks, and in applied social contexts;*
- *be confident in their ability to understand mathematical ideas and concepts including new ones;*
- *have a sense of mathematical self-efficacy, i.e., a confident self-image of themselves as successful in mathematics;*
- *have a sense of personal ownership of mathematics including a sense that they can be creative in Mathematics (Ernest , 2002, p. 12).*

To achieve these goals, the most important factor will probably be the quality of student-teacher relationships in the classroom. Other important factors are firstly learners' success at mathematical tasks over a long time and ownership of this success. Secondly, increasing the cognitive demands in set tasks is vital so that challenge and hence levels of attainment increase. Thirdly, a variety of mathematical tasks and projects should be used to encourage use of initiative and creativity. Fourthly, providing the opportunity for learners to make and express judgements and valued contributions and finally a shift away from individual competitive work towards more group sharing of mathematical ideas are necessary (Ernest, 2002). These suggestions of Ernest to achieve the goals of epistemological empowering are in line with the arguments of Bandura on the courses of improving self-efficacy, as described in section 3.5.1(c). Ernest (2002) claims that epistemological empowerment is the culmination of all the other types of empowerment.

Ernest (2002, p. 13) writes:

It is only when learners are fully empowered mathematically that they will feel they are entitled to be confident in applying mathematical reasoning, judging the correctness of such applications themselves, and critically appreciated by others, across all types of contexts, in school and society.

3.5.4 Ackerman, Kanfer and Beier's findings on APM as domain knowledge

Research by psychologists Ackerman, Kanfer and Beier (2013) provides a link between the role of the non-cognitive and cognitive constructs and domain knowledge in predicting success in Mathematics. They studied 589 undergraduates at the Georgia Institute of Technology, from their first semester through attrition and graduation (up to eight years after their first-semester). The findings from their research indicate two sets of significant and substantial indicators for post-secondary academic success and STEM persistence: (a) a broad set of trait complexes and (b) individual measures of domain knowledge assessed in high school. Together these two sets account for significant variance of university average attrition, and STEM major persistence. "Inclusion of trait-complex composite scores and average AP examination scores raised the prediction variance accounted for in university Grades to 37%, a marked

improvement over traditional prediction measures” (Ackerman et al., 2013, p. 1). They specifically show the potential value of using AP examination scores in the prediction of future performance and STEM major persistence.

Other findings from their research are that the most important predictors of STEM major persistence were receiving credit for AP Calculus and the student having successfully completed three or more AP examinations in the STEM areas. Such students graduated at a substantially higher rate and in fewer semesters of study. The average AP examination score was the single best predictor of academic success after high school GPA. Decisions made early in high school may have a significant impact on later success. AP course-taking and success in AP examinations develop a student’s confidence, which leads to academic success in at least first-year Mathematics (Ackerman et al., 2013).

This research on the importance of domain knowledge supports the findings of several earlier studies. Ma and Johnson (2008, p. 75), proposed that coursework in Calculus during high school “is a powerful filter that critically screens females for prestigious occupations”. This statement is true irrespective of gender. Astin (1984) argued that the most important source of influence at university lies in the development of subject-matter knowledge. Pascarella and Terenzini (2005) argued for the importance of transfer of knowledge in determining the acquisition of new knowledge. Ackerman et al., (2013, p. 5) conclude that domain knowledge is “a potentially important contributing factor in predicting success and retention in STEM fields”. AP examinations provide a good common indicator of this important foundation of prior knowledge and skills at entry to university.

3.5.5 *Synthesis*

The quest to determine the factors predicting success in first-year Mathematics at university has led to the mathematical empowerment theory of Ernest (2002), which provides a holistic view of the empowered mathematical learner.

If learners can acquire skills and significantly master the domain of school mathematics (mathematical empowerment/domain knowledge), and experience examination success

in Mathematics, it will open the gateway to studies in STEM courses (social empowerment). The epistemological empowerment of learners, which involves their development of personal confidence, their sense of mathematical self-efficacy, as well as their sense of personal ownership of and power over Mathematics (Ernest, 2002), will then help them to be successful in their university studies, which could lead to employment opportunities in STEM careers. It is hypothesised that the successful completion of the APM course will be a vital part of this process.

Bandura's theory on self-beliefs (1977), culminating in the recent work of Stankov and Lee (2014) on confidence as the best non-cognitive predictor of achievement in Mathematics, provides insight into what the focus of an APM course should be. Ackermann, Kanfer and Beier (2013) accentuate the role of APM in providing domain knowledge, which together with the construct of self-confidence can lead to success in first-year Mathematics.

The students who have self-confidence in their interest, motivation, and ability when deciding to enrol in AP courses and devote the necessary effort to acquiring domain knowledge in high school are the students that succeed in AP examinations. These students are then also successful in later post-secondary study.

3.6. A CONCEPTUAL FRAMEWORK FOR THIS STUDY

The synthesis as discussed leads to a model for a conceptual framework for this study, shown in Figure 3.1. This model demonstrates a theory on what happens to a learner with self-confidence who enrolls in APM. If he/she finishes the course and writes the exam successfully, this leads to self-efficacy because of the mastery experience in the examination, as well as a good domain knowledge of Mathematics. This leads to confidence in his /her ability to do Mathematics. Consequently he/she will be successful in the NSC Mathematics as well as the NBT tests. With good results in Mathematics he/she will gain access to a STEM course. The self-efficacy, confidence and good domain knowledge leads to success in first year Mathematics and eventually to persistence and graduation. In this process the learner becomes mathematically and socially empowered and thus an epistemologically empowered student.

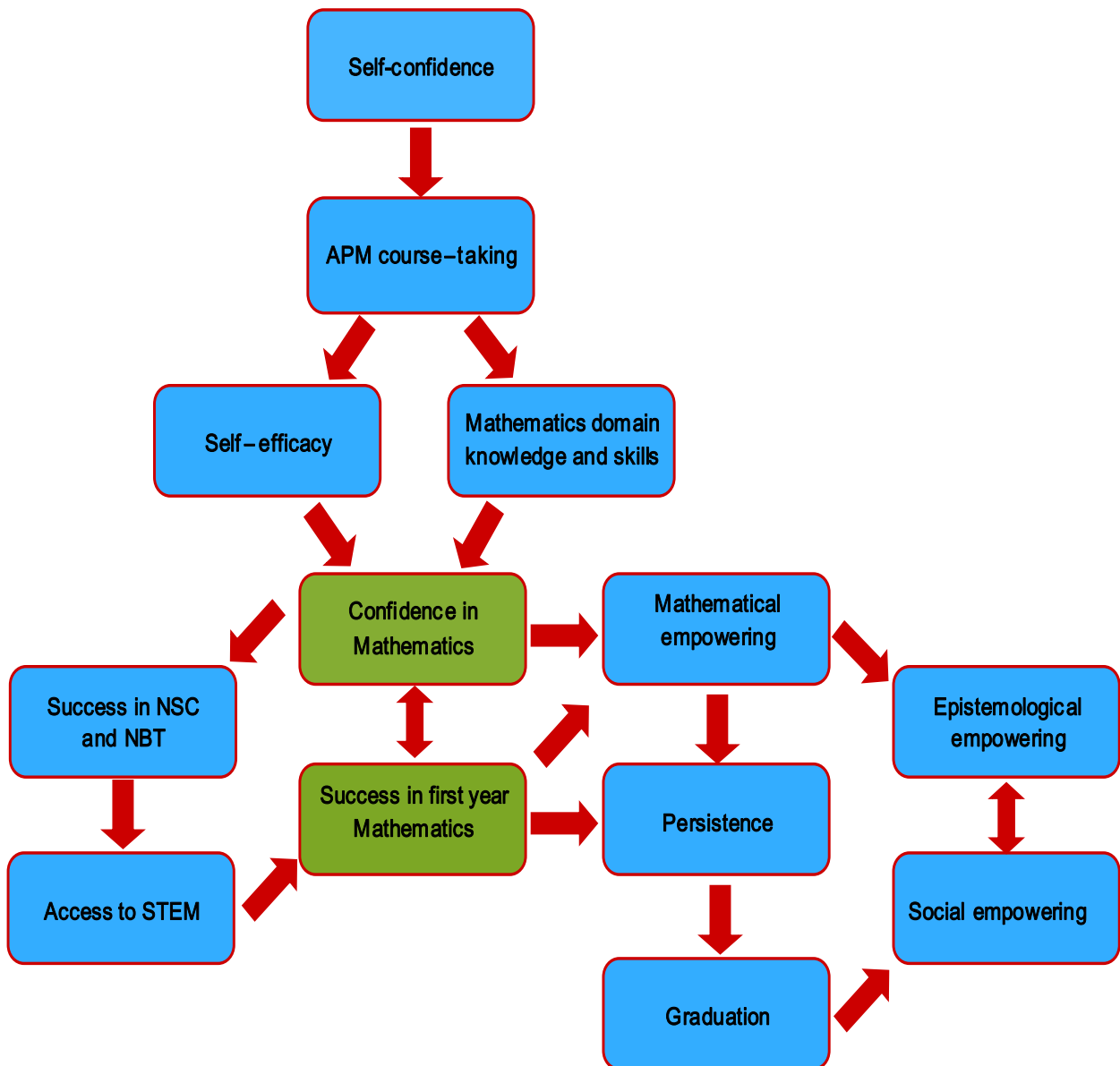


Figure 3.1 Conceptual framework

3.7 SUMMARY

This chapter explicates the different theoretical perspectives underpinning this study. Firstly, the study was positioned within the epistemological phenomenon of education and underpinned by constructivism and the social cognitive theory. Then the notion of student success and some of the most prominent theories on student success were discussed. This was followed by a discussion of Bandura's self-efficacy theory, Ernest's theory on mathematical empowering, Stanlov and Lee's theory on confidence and

Ackerman, Kanfer and Beier's theory on APM as domain knowledge. The chapter ends with a suggested conceptual framework. The next chapter will discuss the methodology used in this study.

CHAPTER 4

RESEARCH METHODOLOGY

4.1 INTRODUCTION

“Research methodology consists of the assumptions, postulates, rules, and methods—the blueprint or roadmap—that researchers employ to render their work open to analysis, critique, replication, repetition, and/or adaptation and to choose research methods” (Given, 2008, p. 516).

In this chapter, the research design and methods used in this empirical investigation are clarified. This chapter is divided into four sections. The first section will discuss the traditional research approaches and paradigms within which research is positioned. It will also describe and justify the choice of this study’s research approach and describe the research paradigm within which this research was conducted. The second section will compare various variables affecting student performance. The instruments and data collection procedures, the sampling method, the data presentation and the analysis procedures used will be discussed. Thirdly, the part of this study in which the questionnaire was used, will be discussed. The chapter will conclude with a summary.

4.2 PLANNING THE RESEARCH

“At its simplest research design is about convincing a wider audience of sceptical people that the conclusions of the research underlying important decisions are as safe as possible” (Gorard, 2013, p. 4). The design is not essentially about techniques and procedures, but about care and attention to detail, motivated by a passion for the safety of research-based conclusions (Gorard, 2013). Key issues such as safety, efficiency and equality were definitely important in this study’s research design. It is even more so in the South Africa of 2014, where a better education is an essential part of a better future for so many people in the country, and where this study is intended to provide insight for future curriculum planning.

Before discussing this study's research approach, it is necessary first to give a short summary of the basic traditional research paradigms or approaches. The researcher did not follow this traditional approach in planning this study, but the discussion thereof will give perspective and provide the background to the researcher's final choice of an approach for this study.

4.2.1 Traditional research approaches and paradigms

Traditionally, there are three recognised designs for conducting research: quantitative, qualitative and mixed methods (Migiro & Magangi, 2011). This, however, seems to be a very simplistic summary of a topic that has been discussed for at least half a century (Mackenzie & Knipe, 2006). In the process of deciding what type of study this is, and of describing the processes involved in undertaking it, it is necessary first to summarise the essence of each of the above-mentioned recognised designs in the next few paragraphs.

The terms 'qualitative' and 'quantitative' are often used in two different contexts, one relating to what is more commonly understood to be the philosophy behind the research, and the other relating to research methods – how data are collected, analysed and reported (Mackenzie & Knipe, 2006). When acknowledging quantitative research as a method, it refers to research that is aimed at testing objective theories, determining facts, undertaking statistical analysis, demonstrating relationships between variables and prediction. The logic used is mainly deductive, as the researcher usually states clearly formulated hypotheses beforehand and knows what type of data needs to be collected. This research approach collects analyses and displays data in numerical form, and the concepts of reliability and validity are of major concern for quantitative research (Creswell, 2013; Gorard, 2013; Van der Merwe, 1996).

Qualitative research is traditionally described as research that aims at the development of theories (inductive logic) and improving understanding of human behaviour and experience. The approach is that of participant involvement and data is usually represented in narrative form (Babbie & Mouton, 2001; Creswell, 2013; Given, 2008). Research questions are formulated to investigate topics in all their complexity (Bogdan & Biklen, 1998). Because the perceptions of the participants direct their actions,

thoughts and feelings, it is necessary to analyse the contexts and meanings they attach to specific processes and events (McMillan & Schumacher, 2006).

When acknowledging that the terms ‘quantitative’ and ‘qualitative’ refer to distinctions about the nature of knowledge, simplistically seen, quantitative research methods are underpinned by the positivist notion of a singular reality, where the only truth that is out there is waiting to be discovered by objective and value-free inquiry (Feilzer, 2010). This reality is referred to as mind-independent and exists even if there are no humans to perceive or experience it (Plowright, 2011).

This is in contrast with the idea that reality is mind-dependent and is “socially constructed through the relationships, psychological activities and shared understandings that we all take part in” (Plowright, 2011, p. 177). Because people live and work in different places, they construct knowledge in different ways and the world consists of multiple realities rather than a single reality. From a social cognitive perspective, knowledge is therefore constructed, not discovered (Given, 2008). Constructivists, using qualitative research methods, favour this view (Creswell & Plano Clark, 2007; Lincoln & Guba, 1994).

This ongoing debate about the link between philosophy and research tends to be polarised between what is referred to as ‘paradigms’ (Given, 2008; Plowright, 2011). Kuhn initially developed and popularised the construct ‘paradigm’ in his book, *The Structure of Scientific Revolutions* (Given, 2008). He described a paradigm as the set of practices that define a scientific discipline at any particular period. Kuhn himself, however, did not consider the concept of paradigm appropriate for the social sciences (Kuhn, 2012). Paradigms have also been described as a loose collection of logically related assumptions and concepts (Bogdan & Biklen, 1998), or the philosophical intent or motivation for undertaking a study (Cohen, Manion & Morrowson, 2000), or sometimes as a theoretical framework (Mertens, 2005). For the purpose of this study, a paradigm can be described as the set of experiences, beliefs and values that affects the way an individual thinks about an issue or topic (Lincoln & Guba, 1994). Traditionally the two fundamentally opposed paradigms are those of positivism or post-positivism and constructivism or the interpretive paradigm (Creswell & Plano Clark, 2007). The

incompatibility of these different paradigms led to the so-called 'paradigm wars', a term coined by Gage (Given, 2008).

The third type of research design, called mixed methods research, has been acknowledged as a response to the paradigm wars (Feizler, 2010). It is evident from a review of the literature that the concept of mixed methodology in research has been engaging scholars worldwide for the past 20 years (Creswell & Garrett, 2008; Creswell & Plano Clark, 2007; Datta, 1994; Greene, 2007; Onwuegbuzie & Johnson, 2006; Teddlie & Tashakkori, 2003). Currently it is still being actively debated (Creswell, 2013; Denzin, 2010; Feizler, 2010; Leesch, Dellinger, Brannagen & Tanaka, 2010).

The different views on what mixed methods constitute, like the terms 'quantitative' and 'qualitative', depend on the focus of researchers. Creswell and Garrett (2008) distinguish between scholars who focus on methods, those who focus on the total process of research, those who focus on the philosophical issues such as the paradigms, or those who just focus on extending existing research designs. Scholars who focus on research methods (such as Creswell, 2013), focus on the 'technique' of research and see mixed methods as the collection, analysis and interpretation of both quantitative and qualitative data in one study. Scholars who see mixed methods as a research process, not just a method, 'mix' quantitative and qualitative research at all stages of the research, from the paradigms directing the inquiry to the final reporting on the study (Teddlie & Tashakkori, 2003). The third group of scholars think about mixed methods as a means of collecting, analysing and using both qualitative and quantitative data within an established approach or research design (approach or methodology), for example ethnography or narrative research. Fourthly, there are scholars that feel that the methods are incidental to inquiry and that the philosophical assumptions are the important focus of inquiry.

Two of the issues this last group of scholars have are whether research methods and paradigms have to fit together (the so-called paradigm/method-fit issue), and what the best paradigm would be for mixed-methods research (best paradigm issue) (Tashakkori & Teddlie, 2010). The paradigm most commonly associated with the mixed methods design is pragmatism (Tashakkori & Teddlie, 2010). Pragmatism means that the focus of research is on the research question and different methods can be employed to

answer this question. Multiple, pluralistic approaches to research are all viable, and the emphasis is on 'what works' (Teddle & Tashakkori, 2003). Under a soft pragmatic paradigm, quantitative and qualitative methods become compatible and researchers could use both in their empirical inquiries (Teddle & Tashakkori, 2003, p. 7). "Pragmatism rejects the idea that we can never arrive at a final and unequivocal understanding of the world and its characteristics. Our beliefs are 'work in progress' and therefore subject to change, amendment and revision. Knowledge and understanding are neither static nor certain" (Plowright, 2011, p 184).

Plowright (2011) also offers an alternative view of the relationship between research and methodology. Traditionally, research methodology is determined by philosophical perspectives and not the other way around. The view that "all knowledge is knowledge from some point of view" (Fishman, as cited by Feilzer, 2010, p. 531) implies that the choice of a social science research method is a reflection of the researcher's epistemological understanding of the world. The interpretation of research findings will expose the researcher's underlying philosophical position, his so-called 'paradigm' (Feilzer, 2010). Plowright's alternative view argues that philosophy does not determine the research methodology employed, but rather that methodology leads to a philosophical perspective that helps explain methodology. Theories are constructed after an event, through the process of induction. Paradigms are in competition with one another to provide an explanation for the methodology of the research (Plowright, 2011).

4.2.2 The research design for this study

In a sense, the researcher therefore forms part of the second group of scholars mentioned in the discussion of the traditional views on a mixed method design, i.e. those scholars who see a research design as a process and not a method. Because the philosophical assumptions are also an important focus of the study, the researcher therefore opted for an 'integrated methodology'. The concept of an integrated methodology is a "new and innovative approach to conceptualising and. thinking about mixed research methodology" (Plowright, 2011, p. 2). Plowright (2011) developed a

framework for this methodology referred to as the Framework for an Integrated Methodology (FralM), which formed the basis of the design of this study.

The ideas in the FralM go beyond the mainstream mixed methods approach to research, and focus specifically on social and educational research. This framework is a model that describes the process of designing, planning and carrying out research. It aims at “supporting the integration of different elements of the research in the process of studying a topic, without favouring a certain element over any other” (Plowright, 2011, p. 4). The traditional terms ‘quantitative’ and ‘qualitative’ are not used at all and the two different types of data are classified as ‘numerical’ and ‘narrative’. ‘Numerical’ data refers to numbers, and is concerned with procedures based on counting and/or measurement, while ‘narrative’ data refers to words and other types of text, and is often ambiguous because it has meanings that can be interpreted in different ways. Narrative data draws on conventional codes of meaning that are based on the use of language. The meanings of the data draw more closely on the subjectivity and interpretations of the researcher (Plowright, 2011), as will be the case in this study.

Plowright refers to the philosophical perspective of this approach as “holistic integrationism” (Plowright, 2011, p. 186). It relies heavily on the paradigm of pragmatism, which argues that the truth is ‘what works’. Working within this framework, the researcher will carry out research that “has a purpose, that is aimed at informing decisions and activities that impact on the world and that solve problems” (Plowright, 2011, p. 185).

The FralM is a combination of a “pragmatic integrated methodology, a relativist social epistemology, a realist social ontology and a realist object ontology” (Plowright, 2011, p. 186). This means that this research focused on the purpose, which was to determine the usefulness of the subject APM, and was driven by the research question and not the researcher’s own philosophical position prior to the beginning of the research. The researcher accepted that what worked in 2013 would change over time and that is a limitation to this research. Figure 4.1 gives a graphical representation of the FralM and shows the design and stages of the research used in this study:

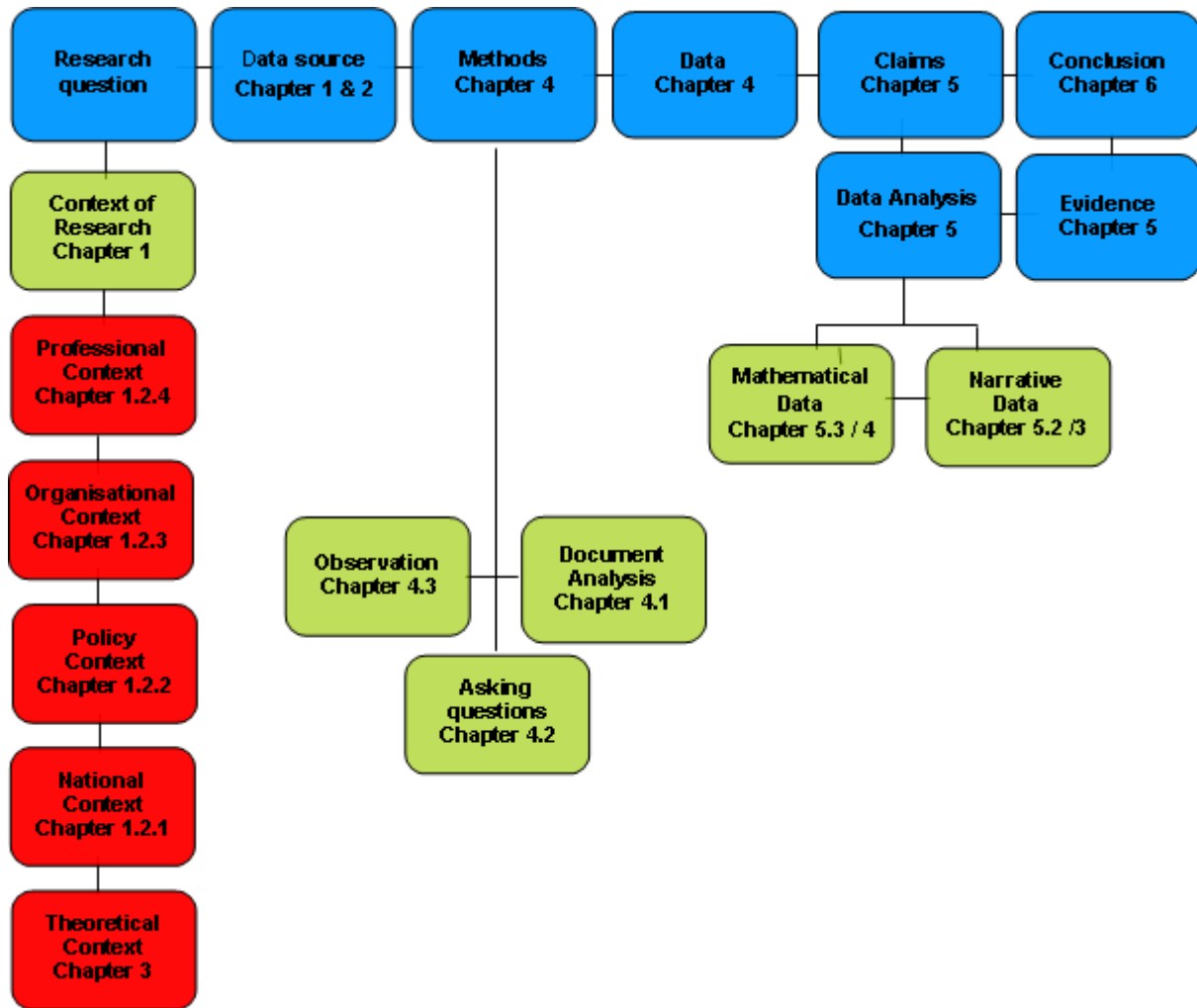


Figure 4.1: The FraIM

Source: Using Mixed Methods: Frameworks for an Integrated Methodology (Plowright, 2011, p. 9).

The starting point of the FraIM was the main research question. When this was known, decisions could be made about the choice of participants, the methods of data collection to be used and methods of data analysis. When the research was under way, the data provided evidence for claims to be made about the participants and this led to conclusions about the research question. The process was therefore not always linear; it sometimes moved from one stage to another and back again, as plans were amended (Plowright, 2011).

4.2.3 The research question

The design of the study started with the following Main Research Question: *To what extent does Advanced Programme Mathematics (APM) prepare learners for the rigour of first-year university Mathematics in the Science, Technology, Engineering, and Mathematics (STEM) programmes?*

In order to provide a complete understanding of this research question, this study was divided into three distinctive sections. In the first section a document analysis of the curricula of the different Mathematics courses was done. The aim was to answer Secondary Research Question A: *How are the APM and the National Senior Certificate (NSC) Mathematics, papers 1, 2 and 3, curriculum related to the first-year Mathematics curriculum at Stellenbosch University (SU)?*

In the second section 'numerical' data was collected with the aim to answer Secondary Research Question B: *To what extent, if any, did the students who took APM in high school and wrote the final examination perform better in their NSC Mathematics National Benchmark Tests (NBT) in Mathematics, and first-year university Mathematics examinations, than those who did not take the subject?*

The third component of the study collected both 'numerical' and 'narrative' data concurrently, analysed it separately and then merged it to form one interpretation of the data with the aim to answer Secondary Research Question C: *What are the student's opinions on, and experience of, the effectiveness of APM in easing the transition between school and university Mathematics?* A detailed description of each of these three components of the study will be given in sections 4.3, 4.4 and 4.5.

4.3 CURRICULAR ANALYSES

4.3.1 The concept

In order to analyse any relationships between the results of prior learning variables such as NSC in Mathematics (papers 1, 2 and 3), the NBT in Mathematics, the APM examination and first-year university Mathematics examinations, it was important to be familiar with the curricula assessed in each of these examinations. For this purpose the

research method of document analysis was used. Document analysis is an analytical method that is used in research on narrative data to gain an understanding of the trends and patterns that emerge from the data (Creswell & Plano Clark, 2007). The analytical procedure entails finding, selecting, appraising (making sense of), and synthesising data contained in documents (Labuschagne, 2003).

Document analysis involves skimming (superficial examination), reading (thorough examination) and interpretation (Bowen, 2009). It requires a systematic and critical examination, rather than a mere description of an instructional document such as a curriculum (Edwards, 2010).

Within the holistic approach, which is the hallmark of the FraIM that underpins this study, the document analysis was used in combination with other research methods, i.e. statistical interpretation of the numerical data and observations made from the narrative data. The researcher attempted to provide a convergence of evidence that breeds 'credibility'. The advantage of examining data collected through different methods was that the researcher could strengthen findings across data sets and thus "reduce the impact of potential biases towards a specific method, a single source or a single investigator's opinion" (Bowen, 2009; Patton, 1990).

4.3.2 *The sample documents*

Plowright (2011, p. 110) describes an artefact, in this study the curriculum, as a "means of encoding or expressing information, knowledge and understanding, in order to make these accessible to and usable by the participants involved in the process". All the artefacts are imbedded in a particular discourse domain, which is located in one or more specific contexts. The artefacts analysed in this study were the curriculum documents on Mathematics in the South African context. These are the National Curriculum Statement for Mathematics, the Independent Examination Board (IEB) curriculum statement for APM, and the curricula for Mathematics 114, Mathematics 115, Mathematics 144 and Mathematics 145 as described in the 2013 yearbook of the Science Faculty at the SU.

4.3.3 Evaluating the evidence

As these curricula all have common characteristics, the researcher performed coding and category construction, based on the data characteristics, to uncover themes pertinent to the curricula (Bowen, 2009).

The approach used for the analyses of the curricula was derived from a revised Bloom's taxonomy of educational objectives (Berger, Bowie & Nyaumwe, 2010; Krathwohl, 2002) and had two dimensions, namely content and cognitive demand.

As an introduction the researcher first gave an overview of the curricula of the NSC Mathematics and APM, as well as the first-year Mathematics 114, 115, 144 and 145 at SU. Then the nature of the two prior learning curricula, i.e. those of NSC in Mathematics and APM, were compared by stating their definitions and looking for similarities and differences. The content of all the different curricula was subsequently summarised to detect any overlaps between the respective curricula and specifically to determine the position of the content of the APM course in relationship to the other courses.

Before analysing the cognitive demands of each curriculum, a comparison between the NSC Mathematics and APM curricula was made in terms of examination mark totals, time allocation and comparability with international curricula. Then the learning outcomes were stated. The verbs used to describe what a learner must be able to do were used as keywords or codes to classify the learning outcomes or objectives into different cognitive levels according to the revised Bloom's taxonomy of educational objective. This taxonomy was represented in a two-way table with the knowledge dimension on the vertical axis of the table and the cognitive process dimension on the horizontal axis. Every learning outcome or objective was classified in one or more cells formed by the intersections of the knowledge and cognitive processes. The cognitive categories were remember, understand, apply, analyse, evaluate and create (Liang & Yuan, 2008), while the knowledge categories were factual knowledge, conceptual knowledge, procedural knowledge and metacognitive knowledge (Krathwohl, 2002). This method clarified the different skills specified in each curriculum.

To identify the skills required in the different examination papers, the taxonomical differentiation of the Grade 12 papers was analysed according to the taxonomy of categories of mathematical demand, as suggested in the 1999 Trends in International Mathematics and Science Study Mathematics survey, which includes four cognitive levels. This was done in order to determine the emphasis of each curriculum in terms of the cognitive levels knowledge, routine procedures, complex procedures and problem solving. The final analysis in this section was an analysis of the verbs used in each of the questions to describe the skills required in the different examinations.

4.3.4 Ethical issues

The researcher aspired to demonstrate objectivity (seeking to represent the outcome of the analysis fairly) and sensitivity (responding to even subtle cues to meaning) in the selection and analysis of data from the curricula (Bowen, 2009). As the subjective interpreter of data contained in the documents, the researcher aspired to make the process of analysis as rigorous and as transparent as possible.

4.4 COMPARISON OF STUDENT PERFORMANCE

4.4.1 Introduction

This section of the analysis process followed a correlation strategy of inquiry. In correlation research, researchers are interested in the degree of relationship between two or more variables (Bruce & Patten, 2009). The students were divided into two groups, i.e. those who took the subject APM in high school and who wrote the final examination, and those who did not take the subject, or who did, but did not write the examination. For the sake of brevity and clarity, these two groups will be referred to as the APM Yes and APM No groups. The results of these two groups of students were compared statistically in order to determine the difference between the two groups with respect to the variables Grade 12 average marks, NBT Mathematics marks, NSC Mathematics marks, first semester university Mathematics marks and second-semester university Mathematics marks. The impact of APM marks as a predictor of success in the first year when used in combination with the other prior learning variables was also determined statistically.

The empirical research objectives pursued in this section of the study were the following:

1. To determine whether the sample was representative of the population
2. To compare the two groups APM Yes and APM No with respect to:
 - a) their demographical variables (gender, type of school, Grade 12 year and ethnicity) when they entered the university
 - b) their prior learning profile when they entered the university
 - c) their first-year performance in Mathematics
 - d) the relationship between their performance in NSC Mathematics and performance in first-year Mathematics
 - e) the relationship between their performance in NBT in Mathematics and first - year performance in Mathematics
3. To determine the significance of APM marks as a predictor of success in first-year Mathematics, when used in combination with the NBT and NSC Mathematics marks.

4.4.1 Participants and sampling

(a) Target population

The target population included all first-year students who registered for specific Mathematics modules at the Department of Mathematical Sciences at SU during 2013. The modules were Mathematics 114 (Calculus), Engineering Mathematics 115 (Introductory Differential and Integral Calculus), Mathematics 144 (Further Calculus and Algebra) and Engineering Mathematics 145 (Further Differential and Integral Calculus). First-year students taking these modules were studying toward bachelor degrees in the STEM programmes. This population did not include students taking the modules Introductory Mathematics 186, Mathematics (Bio) 176 and Introductory Mathematics

(Bio) 124. These modules are either taken by students on the extended degree programme or by students in the biological sciences.

The official application and registration forms of the institution served as sources of biographical information on the target population. The demographic data of the target population in terms of the variables gender, ethnicity, type of school attended and Grade 12 year are summarised using bar diagrams in Figure 4.2. Only the data for enrolment for the modules Mathematics 114 and Mathematics 115 were used, as these modules were pre-requisites for Mathematics 144 and Mathematics 145. For the sake of brevity Mathematics 114 and Mathematics 115 will be abbreviated as MATH 114 and MATH 115 respectively in Figure 4.2:

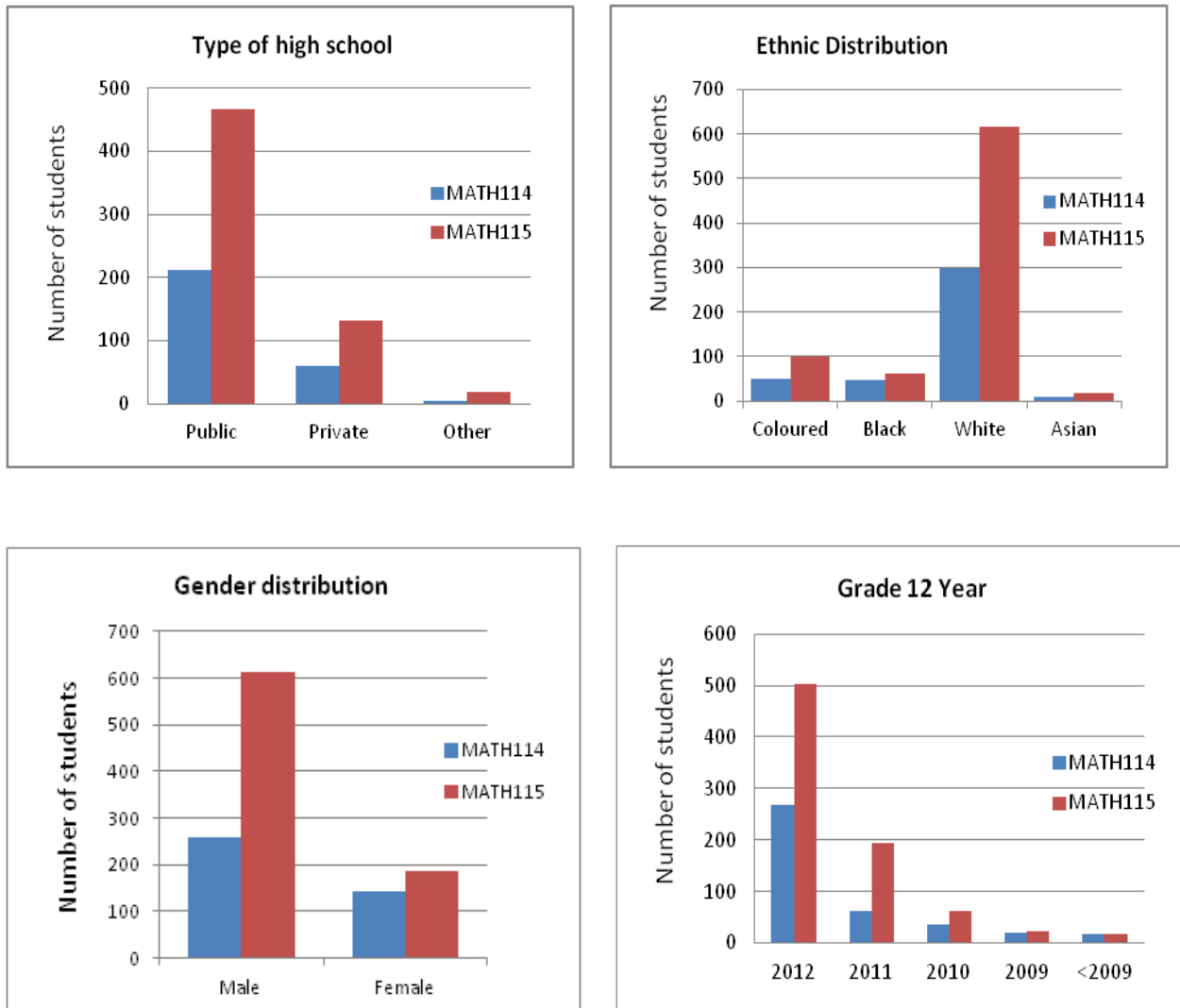


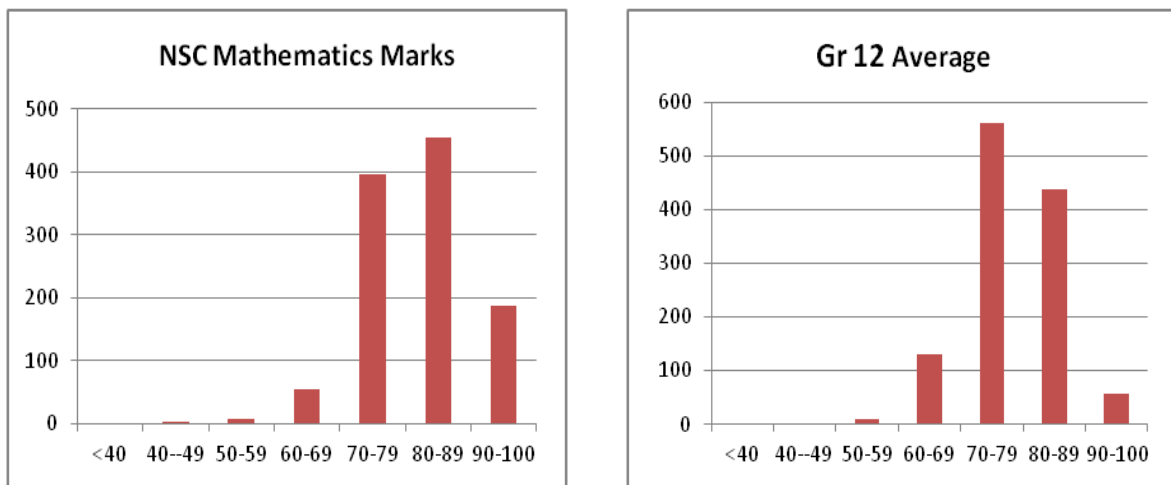
Figure 4.2: Demographic variables of the target population

The following noteworthy observations need to be highlighted:

- The large proportional representation of white students and the exceptionally low representation of Asian students (2%).
- The much higher enrolment for Engineering Mathematics 115 than for Mathematics 114.
- The domination of male enrolment for the Mathematics courses.
- The fact that most of the students enrolled for a university course the year after they had finished high school.

The typical student who enrolled for a course in one of the target Mathematics modules at SU during 2013 could therefore be described as male, white and aspiring to become an engineer.

The next step was to describe the target group in terms of the cognitive variables associated with success in first-year Mathematics. The results of the students in the target population with reference to their Grade 12 average, their NSC Mathematics mark, their NBT Mathematics mark and their NSC Paper 3 mark are summarised in Figure 4.3.



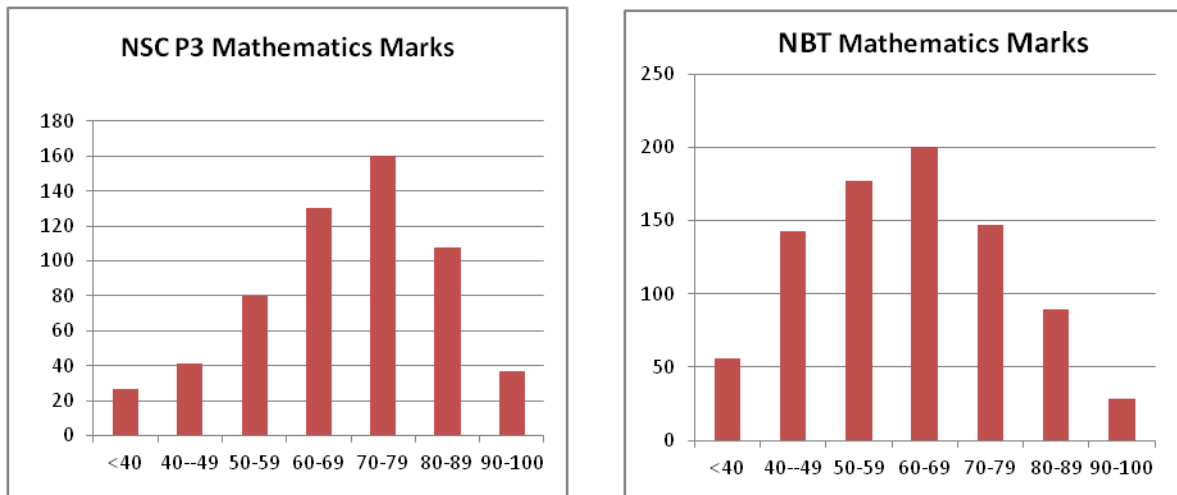


Figure 4.3: Frequency tables of data on cognitive variables of target population

This figure does not distinguish between the students that took Mathematics 114 and those who took Mathematics 115. It is interesting that although the requirement for enrolment in these two courses are a minimum of 70%, there is a small number of students in the target group with a NSC Mathematics mark of less than 70%. The largest mass of students has a Grade 12 average of 70-79%. Although it is not the aim to analyse data in this chapter, the difference in distribution of the marks between the NSC Mathematics and the NBT Mathematics in the target group must be noted.

(b) Sampling collection procedure

The sample consisted of two groups of students: those students in the target population who definitely wrote the APM examination in Grade 12 (APM Yes group) and those who definitely did not (APM No group). This was not a straightforward procedure, as no official records were available. The researcher had to use existing contacts to obtain access to participants and could therefore not select a random sample of participants. This non-probability sampling strategy can be labelled as accidental sampling or convenience sampling, since the researcher had to select those respondents from the population who were obtainable or convenient to reach (Patten & Bruce, 2009).

At SU the student records do not include any reference to the subject APM. Only a student's marks for NSC Mathematics as well as NSC Mathematics paper 3 reflect on

the student record. The researcher firstly asked permission from the IEB to obtain a name list of students that had written the APM examination in 2013, but this permission was not granted. The information therefore had to be obtained from the students themselves.

Then permission was first obtained from the Department of Mathematical Sciences to distribute a list via the lecturers in all the Mathematics 114 and 115 classes during one week in October 2013. Students were asked to indicate whether they had taken APM in high school. The response rate was very low (only 224 or 18.6%); the lecturers attributed this to low class attendance in the run-up to the final examinations. Then the Admaths Centre in Belville that teaches APM in 80 schools nationwide was contacted. They provided the names of 203 students in the 2012 cohort of Grade 12 APM examination candidates. Individual schools provided another 13 names. Lastly a questionnaire was sent to every student in the target population on which the students could indicate whether they had taken APM or not. (Details of this questionnaire are discussed in section 4.4). The number of students that responded to this was 160 (13.5%). The names of all the students that were indicated in the above-mentioned procedures as having taken APM were checked against the target population of all first-year Mathematics students at SU to determine which of these students were enrolled for one of the target Mathematics modules. A total of 436 students had indicated a definite Yes or No for APM. This group of students will be referred to as sample group A.

(c) Sample A

Sample A therefore consisted of students in the target population who definitely wrote the APM examination in Grade 12 and those who definitely did not. The two-way contingency table 4.1 shows the final sample size compared to the size of the target population:

Table 4.1: Sample size and target population size

	Yes APM	No APM	Total:	Total Target Population
Maths 115	115 (14.4%)	237 (29.6%)	352 (44.1%)	797
Maths 114	44 (10.8%)	45 (11.2%)	89 (22.1%)	402
Total	159 (13.2%)	282 (23.5%)	441 (36.7%)	1199

In 2012 the total number of students who wrote the APM examination in South Africa was 1 568. The sample of students in this study who wrote the APM examination represents 9.8% of the total population of APM students in South Africa.

Since the number of students in the Maths 144 group was very small, the researcher decided not to distinguish between the results of Maths 114 and Maths 115 students, but to treat data from both groups as a cohort of first-year Mathematics students' results. This meant that the results of all students who took APM were compared to the results of all the students who definitely did not take APM. The sample then represented 36.7% of the target population. From this point onwards, the terms "first-year Mathematics" will refer to the cohort of students in both Mathematics 114 and Mathematics 115 courses, or in both Mathematics 144 and Mathematics 145. The students were identified by their student numbers to ensure anonymity.

(d) Limitations of sampling

Failure to identify all members of a population of interest (in this case all the students that took APM) can be a source of bias in sampling. Another source of bias in this study's sampling method is volunteerism. The reasons why many of the students in the population chose not to indicate whether they had taken APM were unknown and the researcher could not speculate on the direction in which the given bias would affect the result of the study. The biographical data of the sample group was therefore compared to the biographical data of the target population. The results of this comparison are

shown in Figure 4.4. The variables gender, ethnic distribution, Grade 12 year and school type are shown as a percentage of the total population.

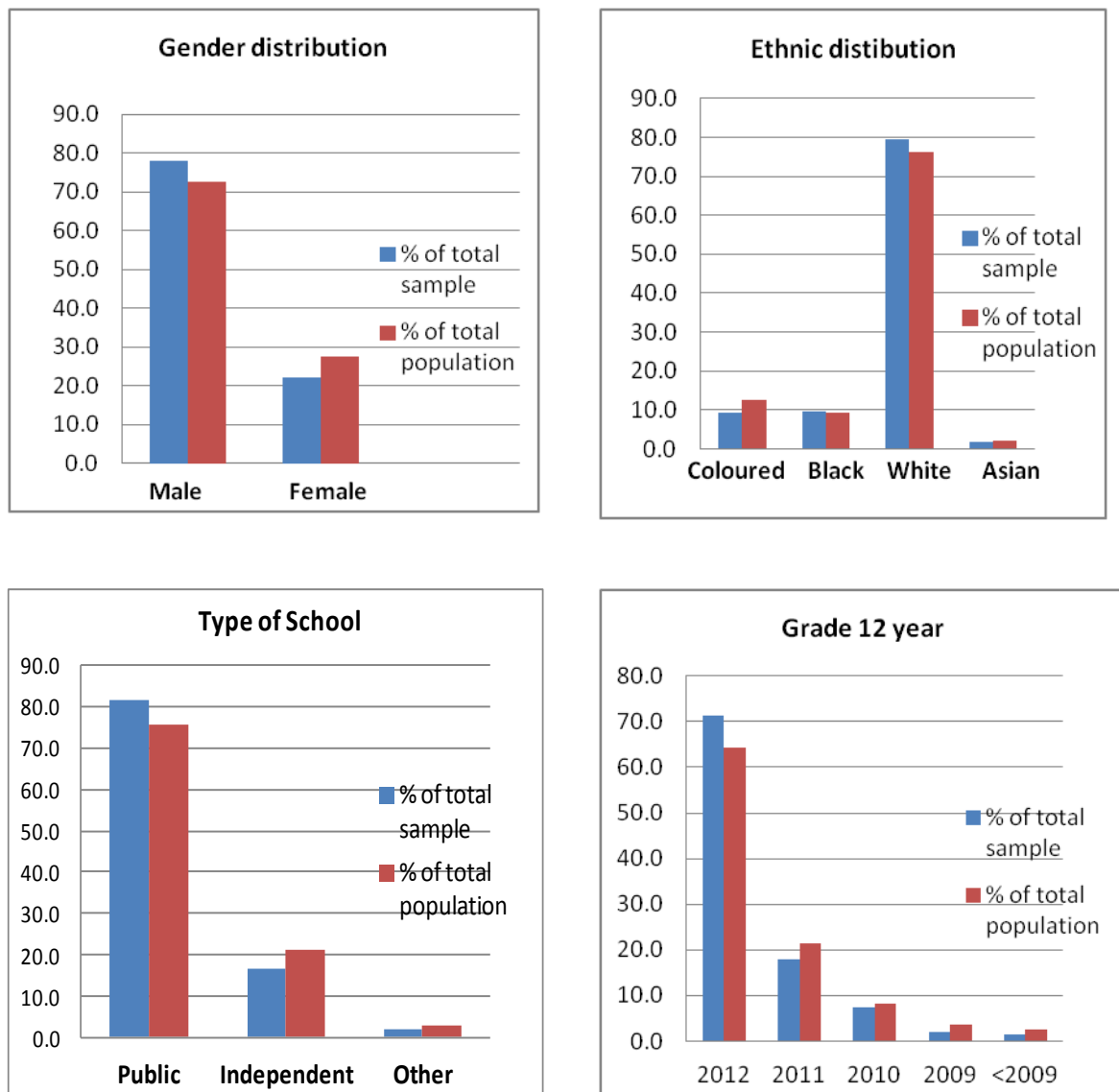


Figure 4.4: Biographical data of sample compared to that of target population

At first glance the bar diagrams in Figure 4.4 show that the biographical data of the convenience sample is representative of the biographical data of the target population on all four biographic variables. However, a chi-square analysis was also done on each of these variables. These results are summarised in tables 4.2- 4.5:

Table 4.2: Results of chi-square test to determine gender representation

Statistic	Chi-square	df	p
Pearson Chi-square	0.1112	df=1	p=0.739
M-L Chi-square	0.1108	df=1	p=0.739

Table 4.3: Results of chi-square test to determine ethnic representation

Statistic	Chi-square	df	p
Pearson Chi-square	21.18	df=3	p=0.0010
M-L Chi-square	23.71	df=3	p=0.00003

Table 4.4: Results of chi-square test to determine matric year representation

Statistic	Chi-square	df	p
Pearson Chi-square	33.93	df=12	p=0.00069
M-L Chi-square	38.82	df=12	p=0.00011

Table 4.5: Results of chi-square test to determine school type representation

Statistic	Chi-square	df	p
Pearson Chi-square	2.329	df=2	p=0.312
M-L Chi-square	2.282	df=2	p=0.320

The results of the Chi-square tests show that the sample population was representative of the target population for the demographical variables ethnicity ($p < .05$), and the year that the student wrote Grade 12 ($p < .05$). The gender distribution ($p = .74$) and type of school ($p = .31$) in the sample did not resemble that of the target population. Any results (except gender distribution and type of school) from the data in the sample are therefore likely to be generalisable to the population of students taking APM at SU.

Although the target population was restricted to first-year students at SU, it is reasonable to expect that the participants in the present study will be similar to students at other universities with respect to gender distribution and final school year. However, ethnical distribution and exposure to school context might differ. The number of students in Sample A that took APM represents 9.8% of the total number of students that took APM in South Africa, so in this regard the findings of the present study are

likely to be of relevance and interest to researchers and practitioners in institutions other than SU.

4.4.2 Research instruments

Apart from the demographic data of the students, five other sets of data of students were used for the first stage of this study:

- The Grade 12 marks in the NSC Mathematics examination.
- The Grade 12 marks in the NSC Mathematics P 3 examination
- The National Benchmark Test in Mathematics mark
- The final examination mark in Mathematics 114 or 115 in June 2013
- The final examination mark in Mathematics 144 or 145 in November 2013
- The Grade 12 mark in APM.

The instruments used were the following: the Gr 12 NSC Mathematics Papers 1, 2 and 3, the NBT Mathematics examination results of 2012, the Mathematics 114, 115, 144 and 145 examination results of SU in 2013. The 2013 papers of Mathematics 114 and Mathematics 115 and 2012 APM Grade 12 examination are provided in Appendix E, as well as website links to the Grade 12 NSC Mathematics Papers 1, 2 and 3. For confidentiality reasons, the NBT Mathematics papers were not available.

4.4.3 Instrument reliability and validity

(a) Grade 12 NSC papers of 2012

The South African Qualifications Authority is the body with overall responsibility for the implementation of the National Qualifications Framework. This framework overarches the whole education and training system in South Africa and sets the unit standards with which all the subjects contained in the NSC Grades 10 – 12 must compare (DBE, 2013).

The Council for Quality Assurance in General and Further Education and Training (UMALUSI), is responsible for overseeing the quality assurance of the NSC. UMALUSI was established under the General and Further Education and Training Quality

Assurance Act (Act No 58 of 2001), to “ensure that continuous enhancement of quality is achieved in the delivery and outcomes of the General and Further Education and Training sectors”. In order for learner achievements to be certifiable, UMALUSI puts extensive measures in place to ensure that the internal assessment, which is carried out at learning sites, and the examinations conducted by the state and private assessment bodies are “fair, reliable and valid” (DBE, 2012, p. 8). The Quality Assurance of Assessment Unit of UMALUSI does this work annually through various moderation and monitoring processes, as the quality assurance body has the responsibility of ensuring consistency of standards from year to year within each assessment body and consistency of standards across assessment authorities (DBE, 2013).

It has to be noted that Higher Education South Africa (HESA), consisting of the vice-chancellors of all the public higher education institutions (HEI's) in South Africa, was sceptical of the predictive validity of the exit qualification of the new curriculum (the NSC). It had not been benchmarked against comparable (international and local) qualifications: “Until the first cohort of NSC learners has completed higher education studies, the predictive validity of this exit qualification remains a promise on paper and not an empirical reality” (Griesel, 2006, p. 12). This was one of the factors that gave rise to the NBT Project (NBTP).

(b) National Benchmark Test in Mathematics

The NBTs were commissioned by HESA in 2005 and reflected almost ten years of research and collaboration among leading content specialists and researchers from HEI's in South Africa. The NBTP is managed by the Alternative Admissions Research Project at the Centre for Higher Education Development at the University of Cape Town. The NBT tests have been explicitly designed to provide a snapshot of the mathematical competencies of test writers at some point in time. The knowledge and skills assessed relate to school mathematical content that is relevant to higher education. The NBT Mathematics tests attempt to determine how well relevant mathematical concepts have been understood and can be applied (Griesel, 2006).

4.4.4 Data collection procedures

All the secondary data used in this part of the research (except the Mathematics marks) were obtained from the SU student administration. The Mathematics marks were obtained from the Mathematical Sciences Department with the permission of the Head of the Department. The Grade 12 results, the results from the access tests, as well as the first and second semester marks in Mathematics, were captured in MS Excel.

4.4.5 Data analysis procedures

This section attempts to provide preliminary answers to the main research question and, more specifically, secondary research question A that guided this study.

Secondary research question A addresses the association between different groups of independent variables, i.e. NSC Gr 12 average, NSC Mathematics mark, NSC paper 3 mark, NBT Mathematics result, and the first- and second-semester marks of first-year Mathematics.

Initially the different variables in the data were classified as nominal, ordinal or continuous, and pivot tables were set up in Microsoft Excel to identify preliminary associations between variables. With the help of the Centre for Statistical Services, the data was analysed by means of a statistical software programme, STATISTICA 12. The basic strategy followed to analyse the data was the following:

First, descriptive statistical methods were used to be able to see the nature of the distribution of the particular variables, and then to be able to identify possible outliers. This data was represented in tables and histograms/bar diagrams. Then the measures of central tendency and dispersion of the particular variables, such as means, medians, modes, quartiles and maximum and minimum values, were described, also using tables and histograms.

Secondly, inferential statistical analysis was done on the variables. When comparing a continuous variable over the different categories of a nominal variable, it was done by using analysis of variance (ANOVA) or the independent *t*-test. In these cases the appropriate F-statistic and the p-value were given. The significance value of the tests

was $\alpha=0.05$. The mean of two groups would differ significantly if the p-value was less than 0.05. When more than two levels of the nominal variables were involved, and the means differed, it was necessary to determine which means were really different. This was done by using a multiple comparisons procedure such as a Bonferroni test. When comparing a nominal variable against another nominal variable, it was done by using a contingency table or cross-tabulation. The assumption was that one variable does not influence the other variable, i.e. they are independent variables. In the case of dependent variables, an appropriate chi-square test such as Pearson's chi-square or the ML chi-square test was done. In this case the p-value was also given. Comparing a continuous variable against other continuous variables was done by using the General Regression model. The aim was to determine if the significance of influence of each of the independent variables on the response variable. The p-values as well as the coefficient of determination β were given. When the two variables were not normally distributed, the Spearman rank correlation analysis was done (StatSoft Inc, 2013).

Table 4.6 gives a summary of the different research objectives associated with secondary research question A, the independent as well as the dependent variables in each analysis, and the statistical analysis method used:

Table 4.6: Summary of the appropriate statistical test for each research objective

	Objective: To determine	Independent variables (IV)	Dependent variable (DV)	Statistical tests
1	If sample is representative of the population			
	Comparison of demographical data of population (all 1 199 students) versus whole sample (Yes + No group) to see if there is a significant difference between the two. (This has been done in section 4.4.1.d)	a) Gender b) Ethnicity c) Type of school d) Grade 12 year	Number of students	Chi -square to determine means and show distribution
2	The demographical profile of the sample group when they entered university			
	Description of the demographical data of the sample APM Yes group versus APM No group	a) Gender b) Ethnicity c) Type of school d) Grade 12 year e) MATH 114 / MATH 115		Chi-square

	Objective: To determine	Independent variables (IV)	Dependent variable (DV)	Statistical tests
3	The prior learning results of the sample when they entered university			
	Description of the prior learning data of the sample APM Yes group versus APM No group (in categories)	a) Gr 12 Average b) NSC Maths results c) NSC Paper 3 results d) NBT results		Independent <i>t</i> -test
4	The Mathematics marks of the sample at university			
	Description of the first-year Mathematics data of the sample (APM Yes group versus APM No group) (in categories)	a) First-semester marks b) Second-semester marks		Independent <i>t</i> -test
5	If there is a correlation between NSC Mathematics marks and university Mathematics marks			
	The difference between the Yes and No groups in relation to performance in the NSC examinations and performance in first semester examination of first-year Mathematics	NSC Mathematics marks	First-semester Mathematics mark	Anova and Bonferonni test
	The difference between the APM Yes and APM No groups in relation to performance in the NSC examinations and performance in the second-semester examination of first-year Mathematics	NSC Mathematics examinations	Second-semester Mathematics mark	Anova and Bonferroni test
6	If there is a correlation between NBT Mathematics marks and university Mathematics marks			
	The difference in performance of the APM Yes and APM No groups in first semester examination of first-year Mathematics for different NBT categories	NBT Mathematics mark	First-semester Mathematics mark	Anova and Bonferroni test
	The difference in performance of the APM Yes and APM No groups in second-semester examination of first-year Mathematics for different NBT categories	NBT Mathematics mark	Second-semester Mathematics mark	Anova and Bonferroni test
7	The impact of each different prior learning variables on performance in first-year Mathematics when they are combined			
	The relative impact of NBT Mathematics marks, NSC Mathematics Marks and APM Mathematics marks on first-semester Mathematics performance	NBT Mathematics marks NSC Mathematics marks APM marks	First-semester Mathematics mark	General Regression Analysis

	Objective: To determine	Independent variables (IV)	Dependent variable (DV)	Statistical tests
	The relative impact of NBT Mathematics marks, NSC Mathematics Marks and APM Mathematics marks on second- semester Mathematics performance	-NBT Mathematics marks -NSC Mathematics marks -APM marks	Second-semester mark	General Regression Analysis
8	The relative impact of NBT Mathematics marks and APM Mathematics marks on second-semester Mathematics performance when first- semester marks are added as independent variable	-NBT Mathematics marks - APM marks -First- semester Mathematics marks	Second-semester mark	General Regression Analysis

Source: Eiselen, 2006, p. 129

4.5 SURVEY QUESTIONNAIRE

4.5.1 Participants and sampling

The second section of the research focused on the students' experience of the influence of APM course-taking on the transition from school to university Mathematics. The researcher set up a questionnaire and conducted a survey of all the students in the target population described in section 4.1. Only the students who completed the questionnaire (n=159) and who obtained a valid first semester mark in Mathematics were used as the sample group. For the sake of clarity, this group will be referred to as sample group B.

The response rate of this survey was 13.4%, which at first sight appears relatively low. However, normally, for large populations, response rates on surveys are only about 5 to 10% (Alreck & Settle, 2004).

Although all students were invited to participate in the survey, from an ethical viewpoint participation remained voluntary, and thus students had the right not to participate. Only a fraction of those who visited the website containing the survey questionnaire was likely to take the time and effort to complete the questionnaire.

This sampling was once again convenient sampling, but each student could respond only once, as student numbers were visible. This sample group may or may not represent the population well and statistical interference is problematic. The relatively

low response rate might also have had the consequence that certain types of participants were likely to be overrepresented and others underrepresented in the sample, which might lead to biased results (Alreck & Settle, 1985).

4.5.2 Research Instruments

The instrument used in this study to collect primary data was an interactive self-reporting online survey questionnaire developed by the researcher. This questionnaire was developed and then administered to a few of the researcher's APM students. It was observed that they understood the questionnaire and their reactions and suggestions were incorporated to help refine it.

The reasons for choosing a questionnaire and not focus groups and interviews were merely of a practical nature. One of the considerations was the nature of the participants – the average Mathematics student would prefer to self-administer a questionnaire in his/her own time in the privacy of his/her home rather than having a face-to-face session with another person. Another reason was the available time of both the researcher and the participants. The timing in the students' academic calendar was another factor. Institutional permission to engage with students was only given in September 2013 and in the following months students were already preparing for an examination and were unlikely to turn up for an interview or focus group. In addition, respondents generally give more honest answers when faced with a computer screen than when faced with an interviewer (Sue & Ritter, 2012).

This survey was posted under the name 'Maths Genius Survey' on the 'e-maties' data website to all the students enrolled for Mathematics 114 and Mathematics 115 during October 2013. The 'e-maties' is the SU's learning management system where lecturers can communicate with students electronically. This survey was submitted with the help of the IT department of SU. Every student received a 'plain text' e-mail message, which cordially requested him/her to complete the questionnaire. They could complete the questionnaire in their own time in the privacy of their homes and then submit it once their responses had been given. The survey was available online for a period of two months.

Table 4.7 summarises the broad constructs, the specific constructs and the level of measurements (Alreck & Settle, 1985) of each item that appeared in the questionnaire. Specific references to the literature review (Chapters Two and Three) that informed the specific constructs in the questionnaire are provided.

Table 4.7: Constructs in the questionnaire

Broad construct	Question number	Specific construct	Measurement scale	Reference in literature review
	1	If a course was repeated in first-year Mathematics	Nominal	
Non-cognitive factors associated with student performance	2	Student's experience of transition from school to university	Ordinal	3.3.3 b
	4	APM course-taking Yes or No	Nominal	
	5a	Confidence level APM provided	Ordinal	3.5.2
	5b	Time management skills APM provided	Ordinal	
	5c	Skill to handle a large workload that APM provided	Ordinal	3.3.1
	6	Class attendance pattern	Ordinal	
	7	Students' experience of APM	Open-ended Question	3.5.3
Cognitive factors associated with previous learning	8	Category in which APM mark fell	Ordinal	
	5d	Understanding Mathematics because of APM	Ordinal	3.4.4
	3	Category in which Grade 12 NSC Mathematics mark fell	Ordinal	
Demographical constructs	9	Student's assessment of APM teacher's knowledge	Ordinal	
	10	Type of high school attended	Nominal	2.6.4
	11	If student would recommend APM	Nominal	

The questionnaire accommodated multiple question formats. There were 11 questions, of which 10 required a single response (check only one). Two of these questions required only yes/no answers and question 7 included a multiple line text area where respondents could answer an open-ended question in their own words. Two of these questions were probe questions where a 'follow-on' response was required. A plain text version of the questionnaire is included in Appendix D.

4.5.3 Method of data collection

The method of data collection used in this section of the study was downloading each individual questionnaire from the 'e-maties' website, and summarising and editing the information on the questionnaires in a format suitable for statistical analysis in a Microsoft Excel spreadsheet. The data obtained from the open-ended question in each questionnaire was printed, copied and pasted on the same sheet in order to simplify the coding process.

4.5.4 Analysis of data

This section is arranged according to the main research question and secondary research question B that guided this part of the study.

Secondary research question B addresses the students' thoughts on, and experience of, the effectiveness of APM in easing the transition between school and university Mathematics. One part of the data obtained by means of the questionnaire was analysed statistically and the other part (open-ended question) by using coding.

Questions 1-6 of the questionnaire were analysed by the Centre for Statistical Analysis using the STATISTICA 12 program. Table 4.8 gives a summary of the different research objectives associated with secondary research question B, the independent as well as the dependent variables in each analysis and the statistical analysis method used.

Table 4.8: Methods used to analyse the research objectives underlying the questionnaire.

Objective: To determine:	Independent variables (IV)	Dependent variable (DV)	Statistical tests
How students experienced the transition from school Mathematics to university Mathematics	APM course-taking		Mann-Whitney U test t -test on ordinal data for descriptive statistics
To what extent APM course-taking led to mathematical confidence	APM course-taking	Confidence measures	Correlation analysis
To what extent APM course-taking helped in handling large workloads.	APM course- taking	Ability to handle large workload	Correlation analysis
To what extent APM course-taking helped with time management	APM course- taking	Improvement in time management	Correlation analysis
To what extent APM course-taking helped with understanding Mathematics	APM course-taking	Improvement in understanding Mathematics	Correlation analysis
To what extent APM course- taking had an influence on first-year class attendance	APM course-taking	Class attendance	Correlation analysis

The questionnaire had one open-ended question (Q 7). Question 7 asked the following: In which other ways did the fact that you took APM in high school have a positive or negative influence on your experience of first-year Mathematics? The answer to this question provided data classified as narrative data according to the FraIM of Plowright (2011). This data was copied into a Word document and then allocated to relevant categories, which allowed for the classification of similar ideas, concepts or themes. Each category was identified by a word or phrase that described the essence of the category. Reassembling the parts again into a coherent whole generated theoretical understanding of the student's experience of APM, as stated by secondary research question B.

Boeije (2009) describes this kind of analysis as forcing the researcher to engage in two activities: thinking and doing. The researcher has to 'do' a lot of things for the findings to 'emerge' from the data, such as "reading, searching, interpreting, writing, conferring, coding and drawing ... she has to think a lot about the data and what they mean by

categorizing data, devising codes and discovering links between categories” (Boeije, 2009, p. 89).

The following preliminary categories (theoretical and conceptual codes) were identified:

- Understanding of Mathematics
- Independent study methods
- Focus on other subjects
- Comprehension of mathematical concepts
- Persistence
- Structure
- Challenges
- Problem-solving skills
- Self-confidence.

According to the FraIM approach to data collection and analysis, the differences between the two types of data, numerical and narrative, are not clear-cut as traditionally argued.

4.6 ETHICAL CONSIDERATIONS

“Ethics is the part of human philosophy concerned with appropriate conduct and virtuous living” (Given, 2008, p. 273).

Ethics in research involves the entire research process, from the nature of the problem being investigated, the reporting of the theoretical framework, the context in which the research is conducted, the data collection instruments and methods used, the participants, the procedures used to analyse the data and the way in which the data is reported (Neuman, 2000).

4.6.1 Ethical issues associated with the research question

From the beginning of research as well as in the data collection phase a clear explanation of what the research was about was given. No changes were made to the original research question after the research had been completed.

4.6.2 Ethical issues associated with the content

The researcher obtained the necessary ethical clearance from the SU ethical committee before the research started (see Appendix A). Permission to access the data and the syllabi of the different modules was given by the SU and the IEB (see Appendices B and C).

4.6.3 Ethical issues associated with the participants

Ethical issues associated with the participants that were addressed in this research were: “informed consent, right of refusal to take part without penalty, right to withdraw without penalty, confidentiality and anonymity, deception and security and safety to prevent any emotional or physical harm” (Plowright, 2011, p.155).

Although respondents to an online survey are always volunteers, the respondents in this study were fully informed of the nature of the research, how the data would be used, the average length of time to complete the survey and that there were no further obligations or risks involved in participating in the survey. This information was provided in the e-mail survey invitation. No signed consent forms for participants were required.

When using the secondary data, only the student numbers were used to identify the students. In the e-mail survey invitation, respondents were assured of confidentiality and anonymity. Technically the responses to e-mail surveys are never truly anonymous because researchers know the respondents’ e-mail addresses. In this study the researcher kept identifying information about respondents separately from their responses.

The researcher did not discuss the research with anyone outside the research milieu and ensured that no individuals could be identified and she was intellectually honest about the source of her ideas in any discussion about the research.

4.6.4 Ethical issues regarding data and data analysis and reporting

The data was kept secure once collected and stored safely to report the results of the research with integrity and accuracy. No values were excluded from the analysis. In this

whole investigation the researcher attempted to maintain objectivity and integrity and to employ professional judgement.

4.7 LIMITATIONS

Limitations to this section of the study arise from the cross-sectional (descriptive) nature of the study, the non-response of students, the validity of the data and the nature of the target population.

The advantages of a cross-sectional study include that it is relatively inexpensive and that it allows researchers to compare many different variables at the same time and to study large groups of individuals more easily. The disadvantage lies in the fact that it cannot account for changes that may occur in the target population over time (Jackson, 2011).

Another concern is whether the sample is representative of the target population. The voluntary participation of the students in the questionnaire implies a certain percentage of non-responsiveness. It cannot with certainty be verified that the resulting sample represents a random and representative sample of the target population, therefore the results of this study may be biased and caution should be applied when generalisation to the target population is made.

4.8 CONCLUSION

In this chapter the research design and appropriate paradigms underpinning this study were discussed. The samples were described, as well as the methods of data analysis. In the next chapter the results of the data analyses will be presented and interpreted and the extent to which demographical and cognitive factors associated with prior learning variables, in particular APM, will influence achievement in Mathematics will be explored.

CHAPTER 5

PRESENTATION AND INTERPRETATION OF RESEARCH RESULTS

5.1 INTRODUCTION

This chapter provides an analysis of the various forms of data in order to arrive at answers to the primary research question, which is:

To what extent does the subject Advanced Programme Mathematics prepare learners for the rigour of first year Mathematics in the STEM university programmes?

In order to do this, three secondary research questions are addressed separately. Secondary research question A is addressed in section 5.2, secondary research question B is addressed in section 5.3 and secondary research question C in section 5.4.

Section 5.2 consists of document analyses, while section 5.3 and the first part of section 5.4 discuss the analyses and interpretations of raw statistical data of the findings on the academic scores of two groups of students in Sample A. One group consists of the students who wrote the Advanced Programme Mathematics (APM) examination in Grade 12 in high school and the other group consists of students who did not write the APM examination. For the sake of clarity and brevity these two groups are referred to as the 'APM Yes' and 'APM No' groups in the data analysis. References to one gender are inclusive of the other gender. In the second part of section 5.4 the views of the students as reflected in their answers to the open question of the questionnaire are analysed and discussed. Comparisons and links are made between different sets of data and the chapter concludes with an integrated summary of the findings of the analyses.

5.2 SECONDARY RESEARCH QUESTION A

The analyses in this section aim to answer secondary research question A, which reads as follows:

How are the Advanced Programme Mathematics and National Senior Certificate Mathematics papers 1, 2 and 3, curriculum related to the first-year Mathematics curriculum (at Stellenbosch University)?

In order to analyse relationships between the performance of students (the achieved curriculum) in APM, National Senior Certificate (NSC) Mathematics and first-year Mathematics examinations, it is necessary to take cognizance of the intended (planned) and the assessed (tested) curricula of these courses, as well as the alignment between these manifestations of the curricula. The differences and similarities of the various curricula and examinations will be determined by using document analysis as described in Chapter 4.3.

5.2.1 Overview of each curriculum

(a) National Curriculum Statement for Mathematics (Grades 10-12)

A new South African school-leaving qualification, the NSC, commonly known as 'matric', replaced the Senior Certificate in 2008. The examinations of this qualification are based on the National Curriculum Statement (NCS), published by the Department of Education (DoE) in 2003. All candidates writing the NSC at the end of Grade 12 must offer either Mathematics or Mathematical Literacy. The National Curriculum Statement for Mathematics (NCSM) is divided into four chapters, each with a specific purpose. Chapter 1 introduces the NCS, outlines its principles and is common to all subject areas. Chapter 2 gives an overview of the field of learning for Mathematics, its definition, purpose and scope, as well as an outline of the specific learning outcomes (LO's). Chapter 3 focuses on the assessment standards (AS's), content and contexts. Chapter 4 focuses on assessment (common to all subject areas) and gives an outline of specific subject competence statements (DoE, 2003).

The Subject Assessment Guidelines (SAG) document, published in January 2008, provides guidelines for assessment in NCS Grades 10 - 12. The guidelines must be read in conjunction with *The National Senior Certificate: A Qualification at Level 4 on the National Qualifications Framework (NQF)* and the relevant Subject Statements. This document indicates certain 'AS's as core and others as optional. Core AS's are examined in Papers 1 and 2 and the optional assessments in Paper 3. Paper 1 focuses on LO 1 and LO 2, while Paper 2 focuses on LO 3 and LO 4. The optional Paper 3 also focuses on LO 3 and LO 4. Topics dealt with in the optional AS's are recursive sequences, Euclidean geometry, descriptive statistics, probability and bivariate data (HESA, 2010). In the latest curriculum document, *Curriculum and Assessment Policy Statement (CAPS)* (DBE, 2011), introduced in Grade 10 in 2012, and to be tested for the first time in Grade 12 in 2014, there is no optional paper anymore and these topics are included in Papers 1 and 2. Topics excluded in this curriculum are Transformations and Linear Programming.

In South Africa there are only two examination boards, the Department of Basic Education (DBE) and the Independent Examination Board (IEB). The examinations of both of these examination boards are based on the South African NCS. Candidates writing either of the two examinations must meet the outcomes and standards set out in the NCS and the examinations are deemed equivalent. The majority of independent schools in South Africa follow the state curriculum and write the same NSC examination as public or state schools. Universities apply exactly the same admission scoring system to school-leavers from the IEB and the public schools, as learners obtain the same qualification, namely the NSC. The IEB learners receive no special recognition or advantage (IEB, n.d.).

(b) Advanced Programme Mathematics

Advanced Programme Mathematics is not one of the official NCS subjects and is examined only by the IEB. Advanced Programme Mathematics is offered as an extension of mathematical knowledge and application. The curriculum statement of the IEB Grades 10 -12 (General) for APM was published in 2006 and describes APM as a "subject in addition to the NSC requirements" on its title page (IEB, 2006, p. 1). This

curriculum document has exactly the same layout format as the NCSM and the respective chapters 1 and 4 are identical. Only the chapters on the LO's, AS's and content differ. APM provides mathematically minded candidates with the opportunity to be challenged and further extend their mathematical competencies (UK National Recognition Information Centre [UK NARIC], 2010). The APM option mostly covers subject areas that are not dealt with by NSC Mathematics, including those subjects covered in the optional third IEB Mathematics paper. The following modules are included in APM:

Module 1: Calculus and Algebra (compulsory subjects covered in the examination)

Module 2: Statistics

Module 3: Finance and Modelling

Module 4: Matrices and Graph Theory

(c) *First-year Mathematics at Stellenbosch University*

The first-year Mathematics curricula of Stellenbosch University (SU) relevant to this study are those of the courses Mathematics 114, 115, 145 and 155. These are the Mathematics courses required for a Bachelor of Science (BSc) degree in Mathematics, the Sciences or Engineering. The curricula of these courses are described in the yearbook of the Faculty of Science (SU, 2013). The Mathematics 114 module, together with Mathematics 144, forms the cornerstone for further study in Mathematics. Science and Commerce students, who require a thorough mathematical grounding for further study in Mathematics and other subjects, require these modules. Similarly, Mathematics 115 and Mathematics 145 form the cornerstone for further study in Engineering.

It is important to note that a mark of at least 70% in the NSC or IEB Mathematics examination is one of the requirements for enrolment in Mathematics 114. A mark of at least 40% in Mathematics 114 is needed for enrolment in Mathematics 145. For Mathematics 115 a mark of at least 70% in the NSC or IEB Mathematics examination is required, and a mark of 40% in Mathematics 115 is a requirement for Mathematics 145. APM and NSC Mathematics Paper 3 are optional subjects and are not used for any admission purposes. A student's National Benchmarks Test (NBT) mark is used for

placement and the Grade 12 Average mark (which needs to be at least 65%) are used as part of the admission criteria at SU (Stellenbosch University [SU], 2013).

5.2.2 Comparison of the nature of the curricula of NSC Mathematics and APM

The curriculum document of the IEB defines APM as “an extension of Mathematics, and based on the same nature of discipline” (IEB, 2006). To verify this claim, the definitions of the two subjects as stated in their respective curriculum documents are compared in Table 5.1.

Table 5.1: Comparison of NSC Mathematics and Advanced Programme Mathematics

NSC Mathematics	Advanced Programme Mathematics
Mathematics enables creative and logical reasoning about problems in the physical and social world and in the context of Mathematics itself.	Advanced Programme Mathematics enhances mathematical creativity and logical reasoning about problems in the physical and social world and in the context of Mathematics itself.
It is a distinctly human activity practised by all cultures.	All Mathematics is a distinctly human activity developed over time as a well-defined system with a growing number of applications in our world.
Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical, and symbolic relationships.	Knowledge in the mathematical sciences is constructed through the establishment of descriptive, numerical, and symbolic relationships.
Mathematics is based on observing patterns; with rigorous logical thinking; this leads to theories of abstract relations.	Advanced Programme Mathematics also observes patterns and relationships, leading to additional conjectures and hypotheses and developing further theories of abstract relations through rigorous logical thinking.
Mathematical problem solving enables us to understand the world and make use of that understanding in our daily lives.	Mathematical problem solving in Advanced Programme Mathematics enables us to understand the world in greater depth and make use of that understanding more extensively in our daily lives.
Mathematics is developed and contested over time through both language and symbols by social interaction and is thus open to change.	The Mathematics presented in Advanced Programme Mathematics has been developed and contested over time through both language and symbols by social interaction, and continues to develop, thus being open to change and growth .

Source: DoE (2003) and IEB (2006)

From Table 5.1 it can be deduced that the nature of NSC Mathematics and APM is basically similar, except for three qualitative differences, as seen in the highlighted

words ‘growth’, ‘greater depth’, ‘additional conjectures and hypotheses’. It appears that APM aims to explore Mathematics in greater depth than the NSC Mathematics. A comparison of the documents on the scope and purpose of NSC Mathematics and APM also shows very strong similarity between the two curricula.

5.2.3 Comparison of content of syllabi

Both the NSC and APM curriculum documents show development coherence in terms of logical progression and hierarchical development of content. The Grade 11 standards build on the Grade 10 standards and the Grade 12 standards build on the Grade 11 standards. Table 5.2 provides a summary of the content of the APM and NSC Mathematics curricula (Grades 10 -12) as described in the third chapter on AS's, content and contexts in each curriculum document, as well as the first-year university Mathematics contents at SU as described in the 2013 yearbook of the Faculty of Science at the SU. The column at the left of the table contains the NSC Mathematics learning outcomes that are used as a starting point.

Table 5.2: Comparison of the content of NSC Mathematics, APM and first-year university Mathematics

		NSC P 1&2	NSC P 3	APM	MATH 114	MATH 115	MATH 144	MATH 145
LO2: Linear Programming		✓						
LO3:Coordinate Geometry		✓						
LO3: Transformations		✓						
LO3:Euclidean eometry			✓					
LO4: Data handling		✓		✓				
LO1:Patterns and sequences		✓		✓				
LO1:Recursive sequences			✓	✓				
LO1:Number and number relationships	Simple and compound Decay	✓		✓				
	Annuities, Bond repayments	✓		✓				

		NSC P 1&2	NSC P 3	APM	MATH 114	MATH 115	MATH 144	MATH 145
	and sinking funds							
	Timelines	✓		✓				
	Model simple population growth and decay problems using discrete two-species Lotka- Volterra predator-prey, discrete logistic and discrete Malthusian population models			✓				
LO4: Probability	Dependent and independent events		✓	✓				
	Mutually exclusive events		✓	✓				
	Exhaustive events		✓	✓				
	Permutations and combinations		✓	✓				
LO4: Statistics	Descriptive statistics	✓		✓				
	Bivariate data		✓	✓				
	Linear regression and correlation		✓	✓				
	Density functions			✓				
	Normal distribution models			✓				
LO3 Trigonometry	Ratios, identities, equations,	✓		✓				
	Compound and double angles	✓		✓				
	Two-dimensional problems	✓		✓				
	Three-dimensional problems	✓		✓				
	Inverse trig and hyperbolic functions						✓	✓
	Radian measure			✓			✓	
	Arc length			✓			✓	
	Area of circle			✓				
	Trig limits			✓			✓	
	Arbitrary powers						✓	

		NSC P 1&2	NSC P 3	APM	MATH 114	MATH 115	MATH 144	MATH 145
LO2: Algebra and equations	Exponents	✓		✓	✓	✓		
	Linear equations	✓		✓				
	Quadratic equations and inequalities	✓		✓	✓	✓		
	Quadratic theory	✓		✓	✓	✓		
	Cubic equations	✓		✓	✓	✓		
	Differential equations	✓		✓			✓	✓
	Parametric equations							✓
	First order linear and separable equations						✓	
	Binomial theorem			✓	✓	✓		
	Mathematical induction			✓		✓		
LO2: Functions and graphs	Number systems	✓		✓	✓	✓		
	Linear, quadratic, hyperbolic, exponential	✓		✓	✓	✓		
	Logarithmic	✓		✓	✓	✓		
	e and ln			✓	✓	✓		
	Absolute value			✓	✓	✓		
	Composite functions			✓	✓	✓		
	Inverse functions			✓	✓	✓	✓	
	Split functions			✓	✓	✓		
	Hyperbolic functions				✓	✓		✓
	Parametric curves						✓	✓
	Inverse hyperbolic and trig functions						✓	
	L'Hospital's Rule						✓	✓
	Polar coordinates and graphs						✓	✓
	Conic sections						✓	✓

		NSC P 1&2	NSC P 3	APM	MATH 114	MATH 115	MATH 144	MATH 145
	Quadratic surfaces and functions in two variables						✓	✓
LO2: Calculus	Limits	✓		✓	✓	✓		
	Limit theorems			✓	✓	✓		
	Continuity			✓	✓	✓		
	First principles	✓		✓	✓	✓		
	Definition of derivative	✓		✓	✓	✓		
	Differentiation rules	✓		✓	✓	✓		
	Applications of the derivative	✓		✓	✓	✓		
	Curve sketching	✓		✓	✓	✓		
	Optimisation	✓		✓	✓	✓		
	Differentiability			✓	✓	✓		
	Implicit differentiation			✓	✓	✓		
	Newton Raphson approximations			✓	✓	✓		
	The mean value theorem			✓	✓	✓	✓	
	Extreme values			✓	✓	✓	✓	
	Definite integrals			✓	✓	✓	✓	✓
Calculus : Integration	Anti-derivative			✓	✓	✓	✓	✓
	Fundamental theorem of Calculus			✓	✓	✓	✓	✓
	U substitutions			✓	✓	✓	✓	✓
	Integration with parts			✓			✓	✓
	Trigonometric integrals and substitution			✓			✓	✓
	Partial fractions and rationalising substitutions			✓			✓	✓
	Other integration techniques			✓			✓	✓
	Riemann sum			✓			✓	✓

		NSC P 1&2	NSC P 3	APM	MATH 114	MATH 115	MATH 144	MATH 145
	Areas and volumes by integration			✓			✓	✓
Complex numbers				✓			✓	✓
Matrix Algebra and applications	Systems of linear equations			✓			✓	✓
	Inverse matrices			✓			✓	✓
	Determinant of matrix			✓			✓	✓
	Matrices to solve graph problems			✓			✓	✓
	Row reduction			✓			✓	✓
	Co-factor expansion			✓			✓	✓
	Cramer's Rule						✓	✓
	Vectors in 2 and 3 dimensions						✓	
Graph theory	Travel problems using graphs and algorithms			✓				
	Minimum connector problems			✓				
	Kruskal and Prim algorithms.			✓				
	Simple travelling salesman problems			✓				
	Nearest-neighbour algorithm			✓				

The following deductions are made from the comparison of the content of syllabi in Table 5.2:

- The focus of the first-year first semester university Mathematics syllabus at SU is Calculus. Although the second-semester syllabus still contains some Calculus, most of the content is not directly related to school Mathematics at all. Some of these topics are Integration, Differential Equations, Polar Equations and Polar Graphs, Conic Sections and Vector Algebra.

- The largest component of the NSC Mathematics syllabus consists of those topics in Mathematics that form the foundations of Calculus, called pre-calculus topics, such as Functions and Graphs, Trigonometry, Algebra and Equations.
- Content done in the NSC syllabus that is not part of the pre-calculus content, or part of the above-mentioned first-year Mathematics courses includes topics such as Coordinate Geometry, Geometry, Euclidean Geometry, Statistics and Financial Mathematics. These topics form the foundation for other first-year courses other than Mathematical Sciences, and teach basic logic and thinking skills necessary at university or in the real world.
- APM teaches pre-calculus topics in Grades 10 and 11 that correspond to those taught in Grade 11 and 12 of the NSC Mathematics syllabus. From the third term of Grade 11, APM teaches Calculus and Matrices content that is very similar to the Calculus and Matrix Algebra content taught in first-year university Mathematics. In addition APM teaches Statistics and Mathematical Modelling that are not part of first-year Mathematics but overlap with similar topics in the NSC curriculum. Students have an optional section of Graph Theory, Mathematical Modelling or Statistics. This content is very useful for students who want to study Computer Programming, Financial Mathematics, Mathematical Modelling or Statistics.
- The content of APM covers a large number of topics and is a mixture of the NSC Mathematics syllabus and the first-year university Mathematics syllabus. It therefore gives a student a feel for Mathematics in the first year.

In the standard generation document the IEB stresses that “it is vital though, that our students who are looking to study anything vaguely mathematical at university, be exposed to concepts, outcomes and standards of advanced Mathematics courses at the school level. We cannot afford to lag behind the rest of the world” (IEB, 2007, p. 4).

5.2.4 Comparison of the content weighting of Calculus in different curricula

As Calculus is the focus of first semester university Mathematics and the branch of Mathematics that overlaps most between NSC Mathematics, APM and first-year university Mathematics, it is interesting to determine the weighting of Calculus in terms

of mark allocation in the examinations and teaching time in class. In the NSC Mathematics examination paper, 35 marks out of the 300 are allocated to Calculus (i.e. 12%) while in the Grade 12 APM examination 120-160 marks out of 300, which is 40% - 53% of the marks total, are for Calculus. If the teaching time spent on Calculus is calculated, it shows that first-year university Mathematics lecturers spend three quarters of the actual teaching time in the first year on teaching Calculus, APM teachers spend half of the teaching time on Calculus and the NSC Mathematics teachers spend 10% of the teaching time on Calculus, which is three to four weeks.

5.2.5 A comparison of examination mark totals and time allocation

Table 5.3 shows the examination marks total and the time allocation for the different examinations:

Table 5.3: Examination mark totals and time allocation

Examination	NSC	NSC Paper 3 (Optional)	APM	Mathematics 114	Mathematics 115
Total marks	P1:150 marks P2:150 marks Total:300 marks	P3:100 marks	P1: 300 marks	P1:50 marks P2:60 marks Total: 110 marks	P1:125 marks
Continuous Assessment	25%	None	None	60%	60 %
Time Allocation for Examination	P1: 3 hours P2: 3 hours Total: 6 hours	P3 :3 hours	P1: 3 hours	P1:2 hours P2: 2 hours Total:4 hours	P1: 3 hours

Source: DoE, (2006) and IEB (2008).

Two major differences between the NSC Mathematics and the APM examinations are the time allocation and the continuous assessment mark (Table 5.3). The NSC Mathematics Papers 1 and 2 requires learners to write 150 marks in three hours (as opposed to 200 marks in three hours in the previous curriculum), while the APM examination allocates three hours for 300 marks. This means that the expected work rate or tempo is twice as fast as in the NSC examination where a learner works at 1.2 marks per minute. From her experience in teaching APM, the researcher has seen that

finishing the final Grade 12 examination in time is a challenge to most APM learners. However, it must be remembered that even though mark allocation is a matter of choice and not necessarily a direct indication of difficulty or tempo, the APM tested curriculum still definitely provides rigorous training in speed and power.

As seen in section 3.3.2(d), Green and Rollnick (2007) argue that time-limited examinations place many restrictions on possibilities for assessing performance at higher cognitive levels. Time constraints, the inherently stressful nature of the testing situation itself and test anxiety are reasons why only the best students (good puzzle solvers) can answer higher-order cognitive skills in an examination (Green & Rollnick (2007). Unfortunately, time limits also teach learners to follow the 'shortest path' and learners then have no time to actually 'do' Mathematics, to think creatively about it, nor to internalize important ideas and concepts (Clark & Lovric, 2009).

Since 2012 the final APM mark in Grade 12 has reflected only the final examination paper's mark as there are no longer any continuous assessment marks. This also attributes to the stressful nature of the final APM examination. To be successful in the final examination, students must not only work independently throughout the year and be confident that they are well prepared, but they must also be familiar with the shortcuts available when using their calculators efficiently, be able to see links between different concepts and be able to handle examination stress.

5.2.6 Cognitive demands

(a) Learning outcomes

In order to compare the cognitive demands of the different curricula, the LO's of the NSC Mathematics, APM, Mathematics 114 and Mathematics 115 curricula are summarised in Table 5.4:

Table 5.4: Learning outcomes of NSC Mathematics APM, Mathematics 114 and Mathematics 115 curricula

NSC Mathematics	Advanced Programme Mathematics	Mathematics 114	Mathematics 115
LO 1: Number and Number Relationships	LO 1: Calculus	LO 1	LO 1
When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions.	The learner is able to establish, define, manipulate, determine and represent the derivative and integral, both as an anti-derivative and as the area under a curve, of various algebraic and trigonometric functions and solve related problems with confidence.	Have a solid theoretical and practical grounding in differential and integral calculus.	Understand the basic concepts of interval notation, absolute values, solving inequalities of real numbers.
LO 2: Functions and Algebra	LO 2: Algebra	LO 2	LO 2
The learner is able to investigate, analyse, describe and represent a wide range of functions and solve related problems.	The learner is able to represent, investigate, analyse, manipulate and prove conjectures about numerical and algebraic relationships and functions, and solve related problems.	Understand the concepts of function, limit, derivative and definite and indefinite integral.	Understand concepts relating to functions of one variable: definition sets, limits of functions, continuity of functions.
LO 3: Space, Shape and Measurement	LO 3: Statistics	LO 3	LO 3
The learner is able to describe, represent, analyse and explain properties of shapes in 2-dimensional and 3-dimensional space with justification .	The learner is able to organise, summarise, analyse and interpret data to identify, formulate and test statistical and probability models, and solve related problems.	Know rules of differentiation and be able to differentiate algebraic and trigonometric functions as well as perform implicit differentiation .	Must be able to demonstrate by means of induction and to handle the binomial theorem.
LO 4: Data Handling and Probability	LO 4: Mathematical modelling	LO 4	LO 4

NSC Mathematics	Advanced Programme Mathematics	Mathematics 114	Mathematics 115
The learner is able to collect, organise, analyse and interpret data to establish statistical and probability models to solve related problems.	The learner is able to investigate, represent and model growth and decay problems using formulae, difference equations and series.	Be able to use differentiation techniques to solve optimisation problems and sketch graphs of functions.	Is able to induce and to apply all differentiation rules and formulas.
	LO 5: Matrices and Graph Theory	LO 5	LO 5
	The learner is able to identify, represent and manipulate discrete variables using graphs and matrices, applying algorithms in modelling finite systems.	Be able to integrate basic algebraic and trigonometric functions.	Know and be able to apply indefinite integration techniques (substitution).
		LO 6	LO 6
		Understand methods of proof and reasoning, including mathematical induction, which are used to establish key results in the development of calculus.	Be able to perform applications of both differentiation and integration.

Source: IEB (2006); DoE (2003); SU (2013)

As described in section 4.3.3, the verbs used to describe the outcomes will henceforth be used as codes to classify the learning outcomes of the NSC Mathematics and APM curricula according to the cognitive levels in the revised Bloom taxonomy, as seen in Table 5.5:

Table 5.5: Learning outcomes of NSC Mathematics and APM curricula according to the cognitive levels in revised Bloom's taxonomy.

Cognitive Knowledge Process for Mathematics 114						
	Remember	Understand	Apply	Analyse	Evaluate	Create
Factual Knowledge						
Conceptual Knowledge		LO 2 LO 1				
Procedural Knowledge	LO 5 LO 4		LO 5 LO 4	LO 3		
Metacognitive Knowledge						
Cognitive Knowledge Process for Mathematics 115						
	Remember	Understand	Apply	Analyse	Evaluate	Create
Factual Knowledge						
Conceptual Knowledge		LO 1 LO 2				
Procedural Knowledge			LO 4 LO 5	LO 3	LO 4	
Metacognitive Knowledge						
Cognitive Knowledge Process for NSC Mathematics						
	Remember	Understand	Apply	Analyse	Evaluate	Create
Factual Knowledge						
Conceptual Knowledge	LO 1	LO 1 LO 3	LO 1 LO 3	LO 1 LO 3	LO 3	
Procedural Knowledge		LO 2 LO 4		LO 2 LO 4	LO 4	
Metacognitive Knowledge						
Cognitive Knowledge Process for AMP						
	Remember	Understand	Apply	Analyse	Evaluate	Create
Factual Knowledge						

Conceptual Knowledge						
Procedural Knowledge	LO 3 LO 5	LO 1 LO 2 LO 4 LO 5	LO 1 LO 2 LO 3 LO 5	LO 1 LO 2 LO 3 LO 4	LO 3 LO 5	LO 3 LO 5
Metacognitive Knowledge						LO 2 LO 3

When analysing the LO's according to the revised Bloom's taxonomy, the following can be deduced:

- The LO's of the courses Mathematics 114 and Mathematics 115 are not described in such a comprehensive way with appropriate use of verbs, as the LO's of the school Mathematics courses. It is therefore difficult to classify the mathematical skills that a student should be able to master at university.
- The skills described in the LOs of the NSC Mathematics concentrate on conceptual and procedural knowledge and the cognitive skills 'Understand', 'Apply', 'Analyse' and 'Evaluate'. The categories 'Create' (also meaning 'Prove' in Mathematics) and 'Metacognitive' knowledge do not feature in this analysis but might be implied by some LO's, although not directly stated according to the verbs used in the description of the LO's. This is in accordance with the following statement of the NSC:

The mastery of Mathematics depends to a large extent on mathematical processes such as investigating patterns, formulating conjectures, arguing for the generality of such conjectures, and formulating links across the domains of Mathematics to enable lateral thinking. Mathematics is a cognitive science. It requires understanding before competence in the Learning Outcomes can be achieved (DoE, 2003, p. 62).

The LO's of APM focus on procedural and metacognitive knowledge and strive to develop all the skills in the taxonomy, from lower cognitive (Remember) to higher cognitive (Create). This observation is in line with the results of the NARIC benchmarking report (2010, p. 88), which states the following on APM:

Advanced Programme Mathematics has been designed with the aim of accommodating more mathematically inclined students, developing candidate maturity, and encouraging candidates to think independently and creatively. The subject is intended to focus more on Mathematics at work. This is reflected in the aims of the examination questions, which seek solutions where students have used both understanding and insight. The focus on such skills appears likely to produce more rounded Mathematics students for entry onto courses at Higher Education level (researcher's highlighting).

In both the NSC and APM curricula, the category 'Analyse' on the cognitive level constitutes the largest proportion, followed by 'Understand' and 'Apply'. The verbs 'Remember' and 'Create' in the LO's of the NSC curriculum have by far the lowest percentage, as is evident in Table 5.5.

UK NARIC (2010, p. 108) claims that, in terms of the level of mathematical competence required:

the APM paper emphasizes more independent and creative thinking on the part of the student, requiring an extension in application of subject knowledge and in turn a more rigorous preparation for the demands of study at undergraduate studies.

(b) Taxonomical differentiation of Grade 12 papers

The taxonomy of categories of mathematical demand, as suggested in the 1999 *Trends in International Mathematics and Science Study* Mathematics survey, includes four cognitive levels. The Grade 12 papers of both the NSC Mathematics and the APM are based on this taxonomy of cognitive levels. The four categories of cognitive levels and their related skills are shown in Table 5.6 below.

Table 5.6: Taxonomical differentiation of Grade 12 question

Taxonomical categories or Cognitive levels	Proportion of the paper		Explanation of skills to be demonstrated
	NSC	APM	
Knowledge	± 25%	10-20 %	<ul style="list-style-type: none"> • Algorithms • Estimation; appropriate rounding of numbers • Theorems • Straight recall • Identifying from data sheet • Simple mathematical facts • Knowledge and use of appropriate vocabulary • Knowledge and use of formulae
Performing routine procedures	± 30%	40-50 %	<ul style="list-style-type: none"> • Problems are not necessarily unfamiliar and can involve the integration of different LO's • Perform well-known procedures • Simple applications and calculations, which must have many steps and may require interpretation from given information • Identifying and manipulating of formulae
Performing complex procedures	± 30%	20- 30%	<ul style="list-style-type: none"> • Problems are mainly unfamiliar and learners are expected to solve them by integrating different LO's • Problems do not have a direct route to the solution but involve: <ul style="list-style-type: none"> -using higher level calculation skills and reasoning to solve problems -mathematical reasoning processes • These problems are not necessarily based on real world contexts and may be abstract, requiring fairly complex procedures in finding the solutions

Taxonomical categories or Cognitive levels	Proportion of the paper		Explanation of skills to be demonstrated
Problem solving	± 15%	5-10%	<ul style="list-style-type: none"> Solving non-routine, unseen problems by demonstrating higher level understanding and cognitive processes Interpreting and extrapolating from solutions obtained by solving problems based in unfamiliar contexts Using higher level cognitive skills and reasoning to solve non-routine problems Being able to break down a problem into its constituent parts – identifying what is required to be solved and then using appropriate methods in solving the problem Non-routine problems based on real contexts

Source: SAG: Mathematics DoE (2008) and APM Assessment Guidelines IEB (2011)

Table 5.6 shows that that the APM places more emphasis on routine problems and less on knowledge reproduction and slightly less emphasis on problem-solving skills than the NSC Mathematics.

(c) Comparison of skills in examination papers

A summary of the skills used in the 2012 NSC Mathematics Paper 1 and 2, APM paper, Mathematics 114 and Mathematics 115 papers, according to the verbs describing what the students should do, is shown in Table 5.7:

Table 5.7: Summary of verbs used to describe skills in different examination papers

Verbs	Number of times the verb occurred			
	NSC Mathematics Paper 1	APM (Only compulsory sections)	Maths 114 Paper 1 and 2	Maths 115
Give	1			
State		1		
Write down	4			
Evaluate	1			

Verbs	Number of times the verb occurred			
Describe	3			
Calculate	9	1		
Identify	1			
Determine/Find	17	18	7	10
Solve	2	1	1	1
Sketch	2	2	1	1
Prove that		1	6	2
Show that		1	2	1
Explain	3	4	1	
Motivate	2	1		
Verify		1		

From this summary of the verbs used in the different examination papers (Table 5.7), it can be concluded that those used to describe the type of skills required in the APM and Mathematics 115 papers are very similar. (Mathematics 115 is the paper for engineering students.) The Mathematics 114 paper focuses on skills such as ‘proving’ and ‘showing that’, which can be described as reasoning skills and are necessary when working with theorems and proofs. These skills are also developed in APM in the section on Mathematical Induction.

The NSC Mathematics paper and the university Mathematics papers cannot really be compared fairly, as there are too many cognitive and pedagogical differences. For example, high school papers are set to provide the opportunity for a large variety of students to fit into different career paths or simply to pass. By contrast, the APM and university Mathematics papers are aimed at mathematically talented students. The difference in format can also be noted. The university papers each consists of a section with multi-choice questions and a section with problems that require a long sequence of logical reasoning. The school papers have no multiple choice questions and the questions are broken up into different steps (stacked) to assist the student in his logical

thinking process. APM introduces students to problem-solving without too many guiding steps. Although there are also ‘Explain’ questions in the NSC papers (two in paper 1 and one in paper 2), these questions only require a short answer of a sentence or two. For example, question 1.4 in the 2012 NSC paper 2 reads: ‘Explain why the observed trend cannot continue indefinitely’, for one mark only.

5.2.8 Summary of Section 5.2

This section attempted to answer the following secondary research question A: *To what extent does the subject Advanced Programme Mathematics prepare learners for the rigour of first year Mathematics in the STEM University programmes?*

The document analyses done on the different curricula show that the content of APM provides a good foundation for first-year Mathematics because it overlaps meaningfully with the first-year Calculus curriculum of the modules Mathematics 114,115,144 and 145. Although the basic aims and objectives of the intended curricula of the NSC Mathematics and APM are very similar, APM also aims to study Mathematics in greater depth than the NSC Mathematics and there are differences in the enacted or taught content and skills, as well as the assessed curricula. The content taught, the amount of teaching time, the time allocation in examinations, the format of the questions and the skills introduced differ substantially. The LO's of APM focus on the development of metacognitive knowledge which is not an explicit focus of the NSC curriculum's LO's. The APM curriculum thus requires an extension in application of subject knowledge.

As discussed in section 3.3.3(c), cognitive difficulties that Mathematics students at university encounter are formal proof, working with theorems, reasoning strategies such as generalizing or working from a definition and a problem-solving approach and thinking style (Bergqvist, 2007; Clark & Lovric, 2009; Craig, 2013). APM introduces the learner to many of these skills, especially those of using formal definitions and proofs, which are not explicitly promoted in the NSC Mathematics curriculum. The APM paper emphasizes more independent and creative thinking and the solutions to questions require both understanding and insight. It appears that APM promotes a deep approach to learning.

It can be concluded that the APM curriculum successfully addresses the transition issues from secondary to university Mathematics as described in the literature review in Chapter 3 and that it provides a rigorous preparation for the demands of study at undergraduate level.

5.3 SECONDARY RESEARCH QUESTION B

This question reads as follows:

To what extent, if any, did the students who took the APM course in high school and who wrote the final examination perform better in the first-year Mathematics examinations than those who did not take the course?

For the sake of clarity and brevity the group of students who took the APM course and wrote the final examination in high school are referred to as the 'APM Yes' group, and the group of students who did not take the APM course are referred to as the 'APM No' group in the data analyses that follow.

Although secondary research question B focuses on the relationship between APM course-taking and performance in first-year Mathematics, examining the relationships between other variables, such as the NSC Mathematics mark, the NBT Mathematics mark, the Grade 12 average mark and the NSC Mathematics Paper 3 mark will provide a helpful context for viewing and interpreting the results of the two groups. These variables are all potential admissions criteria and cognitive factors associated with prior learning. For the sake of this analysis, they are simply referred to as prior learning variables.

The empirical investigation is reported in accordance with the following sub-themes of research objectives:

a) Demographics of Sample A

The descriptive statistics of the two groups at the time when they entered the university with respect to the following prior learning variables:

- NSC Mathematics mark

- NSC Mathematics Paper 3 mark
- Gr 12 Average mark
- NBT Mathematics mark.

The first three variables can be categorised as representative of school performance. The NBT in Mathematics assesses the ability of a learner to combine aspects of prior learning that have a direct impact on the success of first-year university students.

- b) The difference in performance of the APM Yes and APM No groups at university in first-year Mathematics (first and second semesters).
- c) The difference between the APM Yes and APM No group when the following factors are compared:
 - performance in the NSC Mathematics and performance in first- and second-semester examinations of first-year Mathematics
 - performance in Grade 12 average and performance in first- and second-semester examinations of first-year Mathematics
 - performance in the NBT and performance in the first- and second-semester examinations of first-year Mathematics.
- d) NSC Mathematics, APM and NBT marks as predictors of success in first year Mathematics.

5.3.1 Descriptive statistics of the two groups in Sample A when entering university

(a) Demographic profile

Table 5.8 shows the difference between the two groups, APM Yes and APM No, with respect to the demographical variables gender, ethnicity, Grade 12 year and type of school attended.

Table 5.8: Demographic profile of Sample Group A

		N	N	Pearson Chi-Square	Df	P
		APM YES	APM NO			
Gender	Female	36	60	0.11	1	p= 0.74
	Male	123	222			
Ethnicity	Asian	2	5	0.11	3	p<0.01
	Coloured	6	34			
	Black	6	37			
	White	145	206			
Type of school	Independent	32	30	2.33	2	p=0.32
	Public	120	189			
	Other	3	4			
Grade 12 Mathematics	2012	140	176	33.9	12	p<0.05
	2011	14	64			
	2010	4	28			
	2009	1	8			
	<2009	0	6			

The Pearson chi-square test of independence between the nominal variables shows that there is no statistically significant difference between the APM Yes and APM No groups in terms of gender (chi-square with one degree of freedom =0.11, $p=0.74$) and type of school (chi-square=2.33, $p=0.32$). There is, however, a statistically significant difference between the two groups in terms of their ethnicity (chi-square with 3 degrees of freedom=0.11, $p<0.01$) and Grade 12 year (chi-square with 1 degree of freedom =33.93, $p<0.05$). It must be mentioned that the conditions for the applicability of the chi-square test were not met in some of the categories. The categories 'Asian' in 'Ethnicity' and 'Other' in 'Type of School' do not meet this requirement because more than 20 % counts are less than 5 and not every count is larger than 1 (Moore & McCabe, 1989).

Figures 5.1- 5.4 show a graphical representation of the demographical profile of Sample A. As seen in Figure 5.1, the same percentage females and males took APM in high school, but there are significantly fewer females taking the target Mathematics modules at SU, than males.

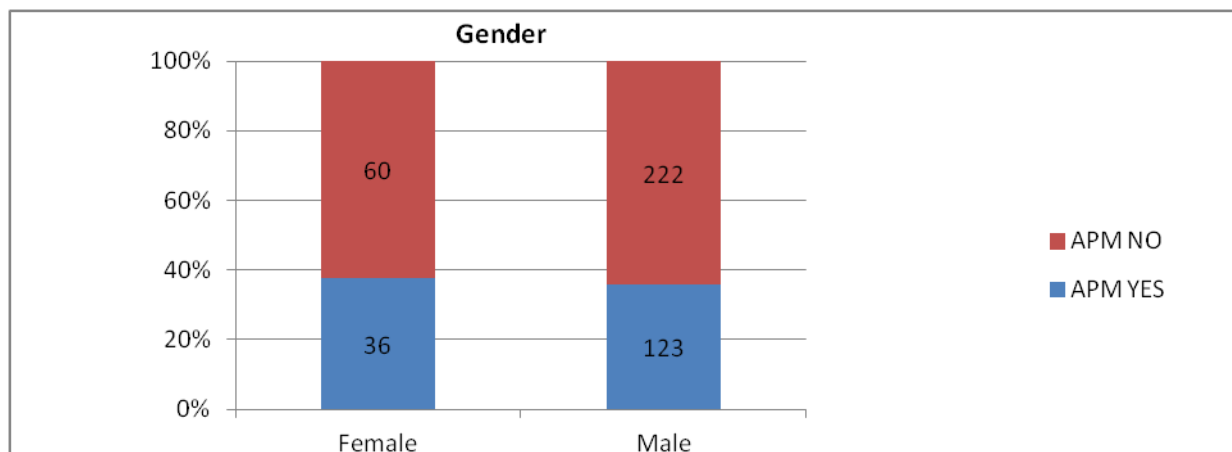


Figure 5.1: Gender distribution between APM Yes and APM No groups

The statistics on the students' final school year show that 88% of the total APM Yes group (140 out of 165) entered university the year after Grade 12, as opposed to the 62% of the APM No group. This shows that a student who enrolled for APM in high school was more likely to enrol for a course in Mathematics directly after high school instead of after a year or two (See Figure 5.2).

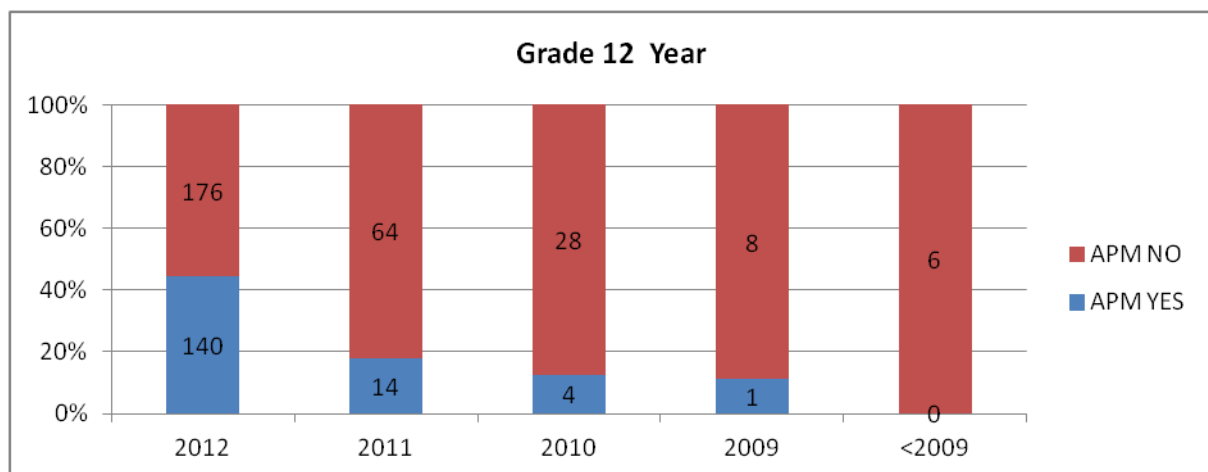


Figure 5.2: Grade 12 year of APM Yes and APM No groups

Student enrolment in 2013 across the different races is shown in Figure 5.3. It is clear that Asian, coloured and black students are not only underrepresented in the target

Mathematics courses at SU, but also in the APM Yes group. This corresponds to the findings on enrolment in AP courses in the USA, as discussed in section 2.6.2 (See Hertberg-Davis, Callahan & Kyburg, 2007; Klopfenstein, 2004). Currently in South Africa, it is still mainly students with higher socio-economic status who have access to APM. This is primarily because of the costs involved in supporting an APM course and the challenge of providing qualified APM teachers. Students within a low socio-economic environment experience lack of adequate preparation in Mathematics and many avoid courses in Mathematics.

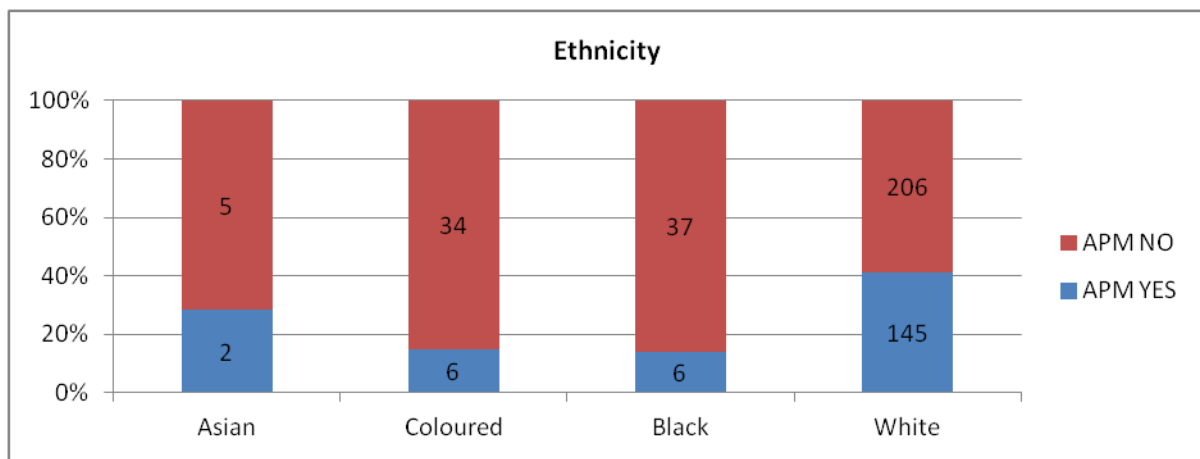


Figure 5.3: Ethnicity of the APM Yes and APM No groups

According to a survey of education in South Africa (SAIRR, 2012), the number of independent schools in South Africa has increased with 87.2% from 2000 to 2011. So too has the number of learners attending independent schools. Despite this rise, independent schools accounted for only 6% of all schools in South Africa in 2011, while learners at independent schools accounted for only 4% of all learners. Figure 5.4 shows the percentage students in each type of school in Sample A. The category 'other' refers to learners doing home schooling.

It appears that a higher percentage of learners in the independent schools take APM (51%), than in the public schools (39%). This can be because many of the independent schools follow the IEB curriculum where APM is often available as a choice subject

during school hours. In public schools, there are no specific teachers appointed for APM, and it is offered after hours.

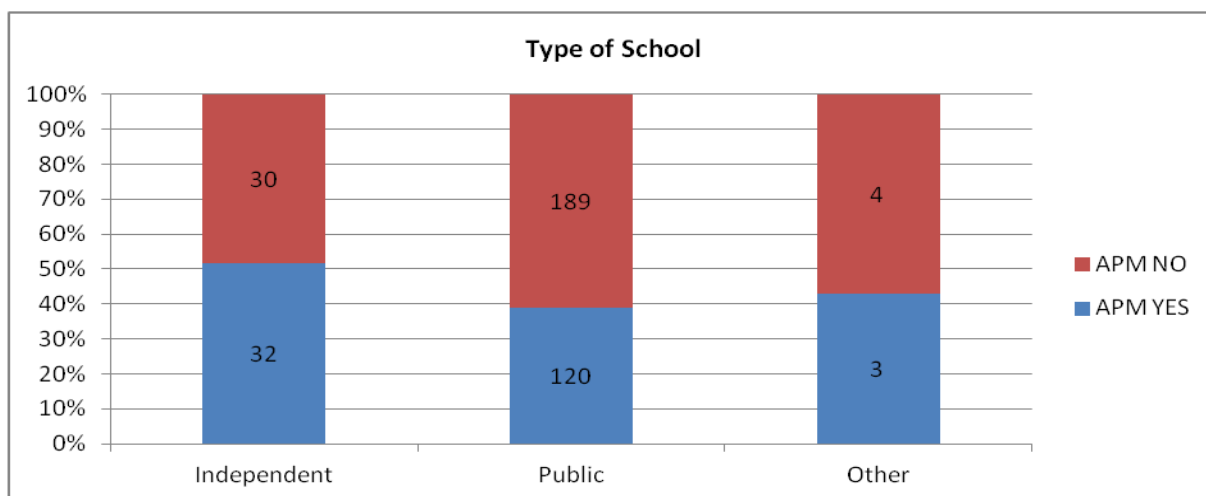


Figure 5.4: Type of high school attended

The analyses of the demographic profile provided valuable information and perspectives on the students participating in the research, but because of the small sample size, for the next analyses, the researcher decided not to distinguish any further results in terms of these demographic variables and to treat the learners as a two cohorts: the APM Yes group and the APM No group.

(b) Prior learning profile

The prior learning results of students entering the university used in this study are their Grade 12 average mark, their NSC Mathematics mark, their NSC Mathematics Paper 3 mark (only if it was a subject of choice) and their NBT Mathematics mark.

Table 5.9 shows the results of two-sample *t*-tests (ANOVA) for each of these four variables. This test gives the mean of the APM Yes and APM No groups with the standard errors (i.e. the common standard deviation of a group divided by the square root of the sample size), as well as the lower and upper limits of the 95% confidence intervals. The mean provides information about the central tendency of the variable. The confidence intervals give a range of values around the mean where it can be expected

that the 'true' (population) mean is located with a given level of certainty (Hill & Lewicki, 2012).

Table 5.9: The mean and confidence intervals for the previous learning variables

Variable	Sample Group	Mean	Std Err	-95,00%	+95,00%	N	P
Grade 12 Average Mark	APM Yes	81.79	0.53	80.74	82.84	159	p<0.01
	APM No	78.09	0.40	77.30	78.88	281	
NSC Mathematics Mark	APM Yes	87.25	0.66	79.44	81.43	152	p<0.01
	APM No	80.43	0.51	85.95	88.55	260	
NBT Mathematics Mark	APM Yes	70.87	1.21	68.50	73.25	141	p<0.01
	APM No	59.32	1,03	57.29	61.36	192	
NSC Mathematics P 3 Mark	APM Yes	75.44	1.26	72.95	77.93	109	p<0.01
	APM No	69.29	1.21	72.95	77.93	119	

The results indicate that there is a statistically significant difference between the mean marks of the APM Yes and APM No groups on the Grade 12 average mark ($p<0.01$), the NSC Mathematics mark ($p<0.01$), the NSC Paper 3 mark ($p<0.01$) and the NBT Mathematics mark ($p<0.01$). In each case, the mean of the APM Yes group is statistically significantly higher than that of the APM No group.

Because there was such a large difference between the means of the two groups APM Yes and APM No across the different variables, it is necessary to look not only at the average performance in university Mathematics, but also at the performance per category of the prior learning variables. The data on these variables are divided into the following performance categories: < 40%; 40-49%; 50-59%; 60-69%; 70-79%, 80-89% and 90-100%. A comparison of the percentage of the total number of students in both groups over the different categories is represented in bar graphs in Figures 5.5 to 5.8. The differences in means over the different categories can be seen clearly.

Figure 5.5 shows the distribution of the Grade 12 average marks over the different categories. The distribution of data for APM No is right skewed, which means that the mass of the distribution of marks is on the left of the figure. The distribution of the marks of the APM Yes group is left-skewed, which means exactly the opposite. The modal class of the APM Yes is the 80-90% category, which means that the largest number of students performed in this category. The modal class of the APM No group is a category lower, i.e. the 70-79 % category.

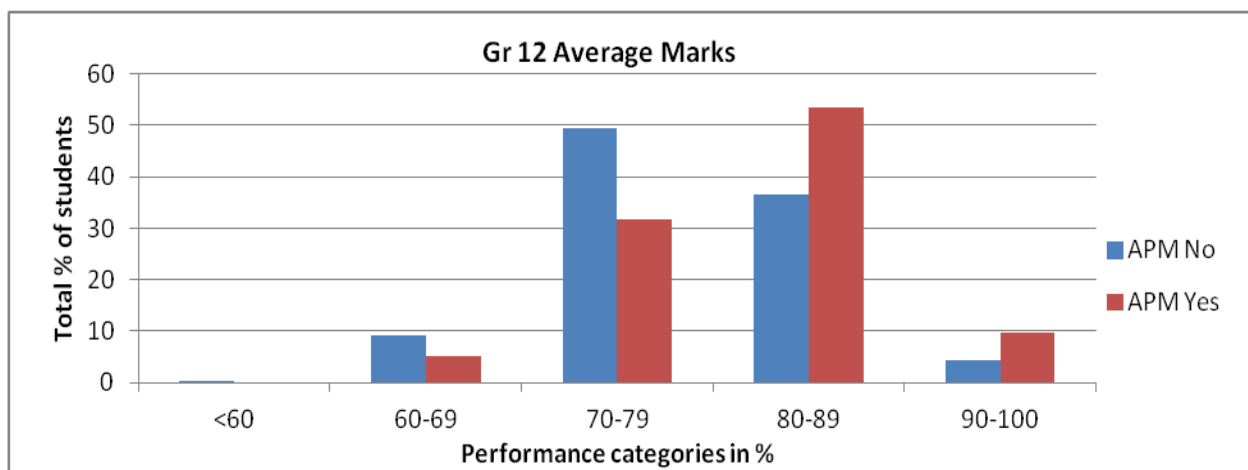


Figure 5.5: Grade 12 average marks across different performance categories as a percentage of Sample A

In Figure 5.6 the distribution of NSC Mathematics over the different performance categories is shown. The marks of APM Yes group are left-skewed, which means that the majority of students perform in the categories 70-79%, 80-89% and 90-100%. This corresponds with the minimum requirement of 70% for enrolment in the target Mathematics modules at SU. The modal class of the APM No group is also a percentage group lower than that of the APM Yes group, which is in the category 90-100%.

It is important to note that the APM Yes group had very high marks in NSC Mathematics when they entered the university. This must be interpreted acknowledging the concern of many universities that school-leaving marks are no longer reliable signals of

preparedness and academic potential as discussed in section 1.2.2(c). (Schöer, Ntuli, Rankin, Sebastiao & Hunt, 2010).

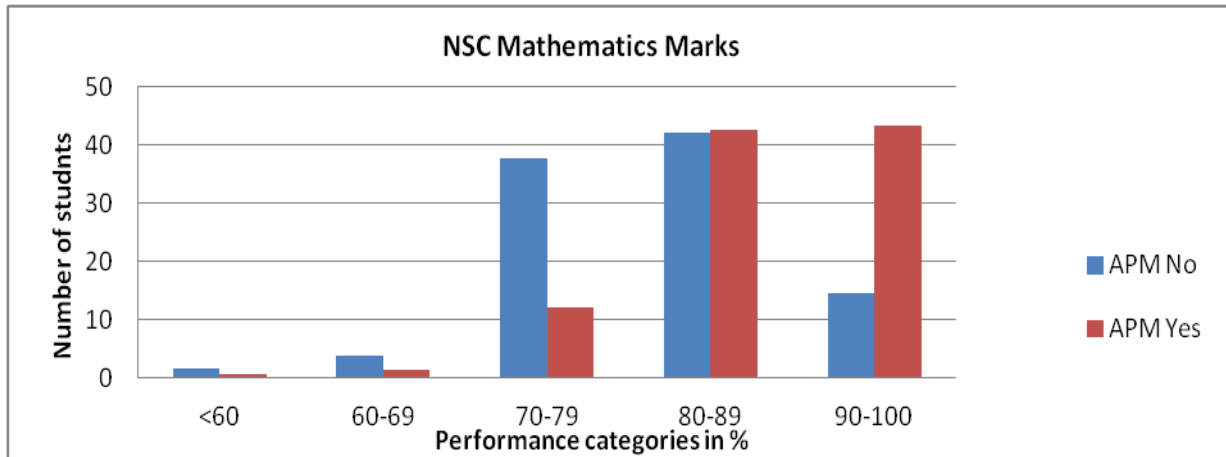


Figure 5.6: NSC Mathematics marks across different performance categories as percentage of Sample A

The NSC Mathematics Paper 3 is an optional paper on Geometry, Probability, Bivariate data and Recursive sequences, and 51% of the sample target group wrote this paper. In 2012 only 22% of the total number of students in South Africa who wrote Paper 3 obtained more than 70% for this paper and only 9% more than 80% (DBE, 2013). In this analysis of NSC Mathematics Paper 3 marks (See Figure 5.7), the APM Yes group is skewed to the left with approximately 80 % of the APM Yes group's marks higher than 80%. The largest percentage of students in the APM No group is performing in the 70-79 % categories. The fact that the NSC Paper 3 results are weaker than those of the NSC Papers 1 and 2 supports the opinion of Jacobs (2010) that the third paper requires more real problem-solving skills than the other two papers.

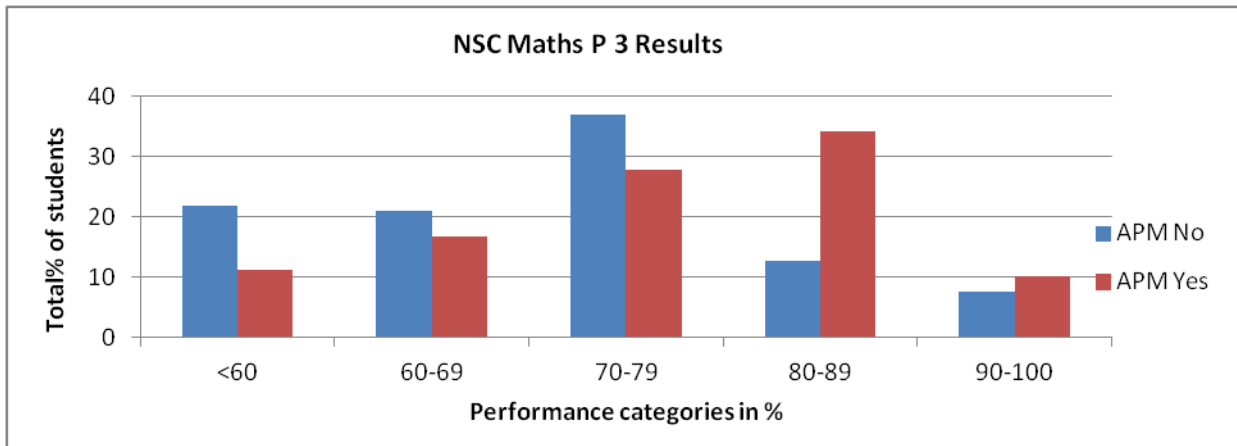


Figure 5.7: Mathematics Paper 3 marks across different performance categories as percentage of Sample A

The distribution of the results of an alternative admissions test, the NBT Mathematics, is shown in Figure 5.8. These tests aim to determine “potential to learn when in an optimum environment” (De Beer & Van der Merwe, 2006, p. 547).

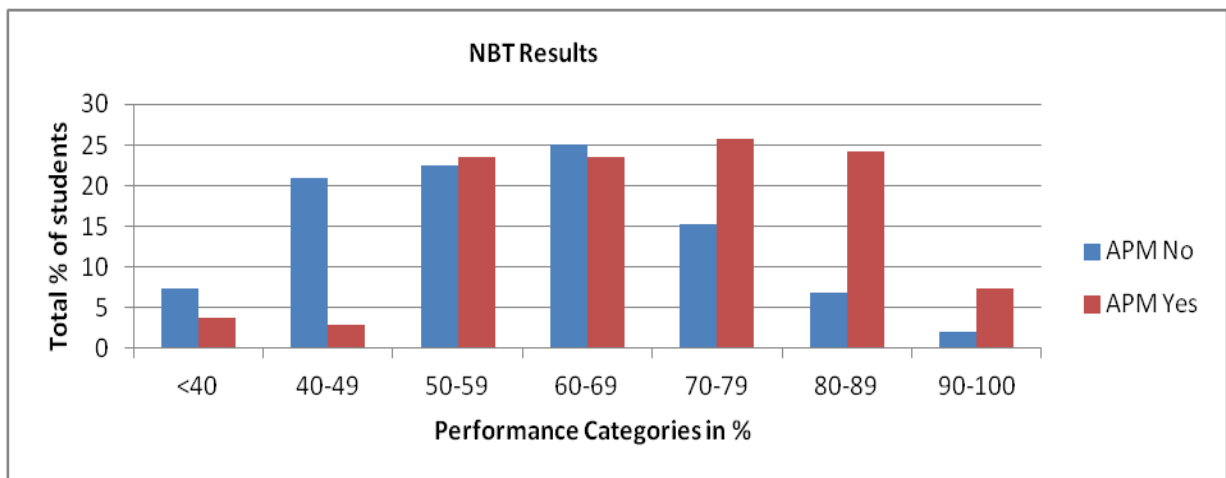


Figure 5.8: NBT Mathematics marks across the different performance categories as a percentage of Sample A

In Figure 5.8 the bar graph shows a fairly normal distribution of the NBT Mathematics marks of both the groups. Both groups are approximately symmetrical around the mean, in contrast to the other three prior learning variables discussed above. As seen in Table 5.9, the means of the both groups are considerably lower in the NBT tests than in the NSC Mathematics, and the modal class of the APM Yes group is again higher than that

of APM No group, which supports the necessity of an alternative admission signal as discussed in section 3.3.2(b).

When observing these frequency distributions it is clear that when entering the university, the APM Yes group was already a group with higher marks in the four cognitive variables representing prior learning than the APM No group.

Many studies cited as justification of positive effects of the AP programme in the US, often ignore the role of student self-selection. As discussed in section 2.6.6(d), Klopfenstein and Thomas (2010) raised the question of whether the better performance of the APM Yes group could be because they are students with naturally higher abilities and more motivation. They caution that researchers must distinguish between prediction and causality concerning APM course-taking. The results of this study confirm their findings that AP course-taking alone may be predictive of university success.

Because there was a significant difference between performance in the APM Yes and APM No group in the prior learning variables, it is not sufficient to compare only the means of the different prior learning variables, but for the sake of completeness it is also necessary to analyse their average performance in the first year of university Mathematics.

5.3.2 Descriptive statistics of APM Yes group and APM No group in Sample A during the first year of university

An independent sample *t*-test is conducted to determine the difference in performance for the APM Yes and the APM No group in the first and second semester of first-year university Mathematics. In this test it is assumed that the dependent variables are normally distributed against the two independent groups. First semester marks referred to are marks in the modules Mathematics 114 and Mathematics 115 and second-semester marks are marks in the modules Mathematics 144 and 145.

Table 5.10: The mean and confidence intervals for first-year Mathematics

Variable	Sample Group	Mean	Std Error	-95,00%	+95,00%	N	p
First-semester Mathematics	APM YES	69.99	1.024	67.98	71.00	157	p<0.01
	APM NO	60.85	0.781	59.32	62.39	270	
Second-semester Mathematics	APM Yes	65.25	1.154	62.98	67.52	142	p<0.01
	APM NO	58.86	0.913	57.06	60.52	227	

There is a statistically significant difference between the means of the APM Yes group and the APM No group in the first semester ($p<0.01$) as well as the second semester of first-year Mathematics ($p<0.01$). This is to be expected, since the APM Yes group already entered the university with better marks than the APM No group. However, the gap between the two groups narrowed in the second semester.

Further analyses are necessary to determine the categories in which the means differed. Figure 5.9 and Figure 5.10 expand on the information in Table 5.10 by showing the difference between the frequency distribution of the marks in the two groups over the different performance categories in first- and second-semester Mathematics.

In Figure 5.9 and Figure 5.10 the category 'No Mark' represents the data of students who did not write the examination. This may either be because the minimum requirements were not met, or because the students were unable to write the examination. It can also be because the students had done the course in a previous year. Students obtaining an average semester mark of 40% or higher are admitted to the examination of the Mathematics modules. The final mark is determined in a 2:3 ratio between the semester-mark and the examination or supplementary examination mark. Supplementary examinations are granted to students obtaining a final mark of 42% to 49%. A mark of 50% is required to pass the module (SU, 2013), and as seen in Figure 5.9 about 7 % of the APM No group and 3% of APM Yes failed the first-semester Mathematics. The distribution of the first-semester Mathematics marks of the APM No group is skew to the right, with the mass of APM No students performing in the 50-59%

category. The marks of the APM Yes show a normal distribution around the mean of 69.99% (See Tabel 5.10).

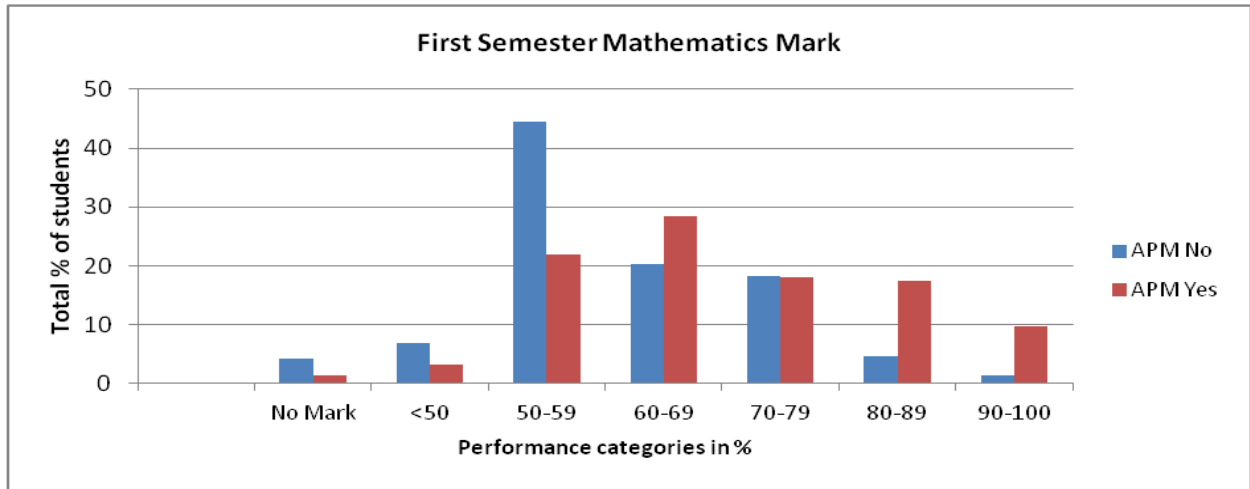


Figure 5.9: First-semester Mathematics marks across different performance categories as percentage of Sample A

In the second semester there was a higher number of failures in both groups compared to the first semester (Figure 5.10) and the mean of the APM Yes group was 5% lower than in the first semester (Tabel 5.10).

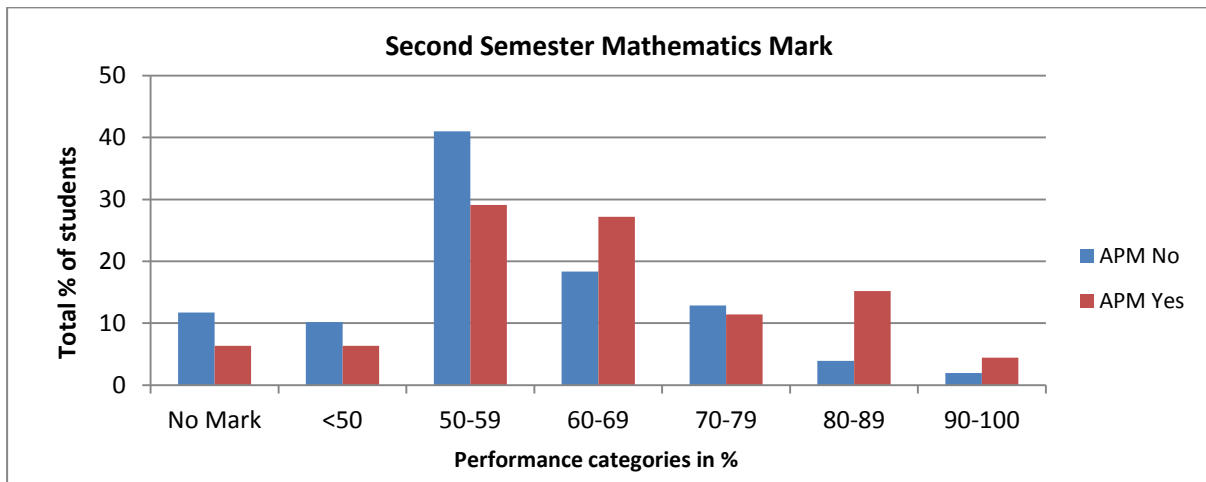


Figure 5.10: Second-semester Mathematics marks across different performance categories as percentage of Sample A

Since it has been determined that there is a significant difference between the performance of the two groups on the different prior learning variables, it can be expected that there would be a difference in performance in the first and second semesters over the different categories. Hence an analysis of the marks of the two groups across the different categories of their NSC Mathematics marks (Tables 5.11 and 5.12 and Figures 5.11 and 5.12) and NBT Mathematics marks (Tables 5.13 and 5.14 and Figures 5.13 and 5.14) was done. A two-way factorial design ANOVA was used to determine if there were a significant difference in performance between the APM yes and no groups for the different performance levels of the prior learning variables. Then a multiple comparisons procedure, such as a Bonferroni test, was used to see in which categories the means show a significant difference.

(a) First-semester Mathematics across NSC Mathematics categories

Table 5.11: Anova and Bonferroni tests for difference in first-semester Mathematics performance across NSC Mathematics categories

Effect	Univariate Tests of Significance for 1 st Semester % (DATA 20140726 in Grafik 20140826.stw)				
	SS	Degr. of Freedom	MS	F	p
Intercept	182388.9	1	182388.9	1738.287	0.000000
NSC Maths mark categories	15333.4	4	3833.4	36.534	0.000000
APM course-taking 1	29.1	1	29.1	0.278	0.598572
NSC Maths mark categories*APM course-taking 1	2164.0	4	541.0	5.156	0.000482
Error	35464.5	338	104.9		

Bonferroni test; variable 1 st Semester % (DATA 20140726 in Grafik 20140826.stw)												
Probabilities for Post Hoc Tests												
Error: Between MS = 104.92, df = 338.00												
Cell No.	APM course-taking 1	NSC Maths mark categories	{1} 56.904	{2} 60.394	{3} 65.375	{4} 70.765	{5} 62.333	{6} 55.833	{7} 64.426	{8} 47.500	{9} 82.130	{10} 58.000
2	No	80-89	0.954		1.000	0.000	1.000	1.000	1.000	1.000	0.000	1.000
3	No	60-69	1.000	1.000		1.000	1.000	1.000	1.000	1.000	0.001	1.000
4	No	90-100	0.000	0.000	1.000		1.000	0.001	0.284	0.088	0.000	1.000
5	No	<60	1.000	1.000	1.000	1.000		1.000	1.000	1.000	0.056	1.000
6	Yes	70-79	1.000	1.000	1.000	0.001	1.000		0.446	1.000	0.000	1.000
7	Yes	80-89	0.000	1.000	1.000	0.284	1.000	0.446		1.000	0.000	1.000
8	Yes	60-69	1.000	1.000	1.000	0.088	1.000	1.000	1.000		0.000	1.000
9	Yes	90-100	0.000	0.000	0.001	0.000	0.056	0.000	0.000	0.000		0.906
10	Yes	<60	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.906	

There is a significant interaction effect ($p=0.000482$) between the APM course-taking and the NSC Mathematics mark categories, meaning that the difference in performance in first semester between APM Yes and No categories are not constant for all the NSC Mathematics categories.

The Bonferroni test (Table 5.11) shows that there is a significant difference between the means of the first-semester Mathematics marks of the APM Yes group and the APM No group in the NSC Mathematics 90-100% category, but not in the category 80-89%. Figure 5.11 gives a graphical representation of these results.

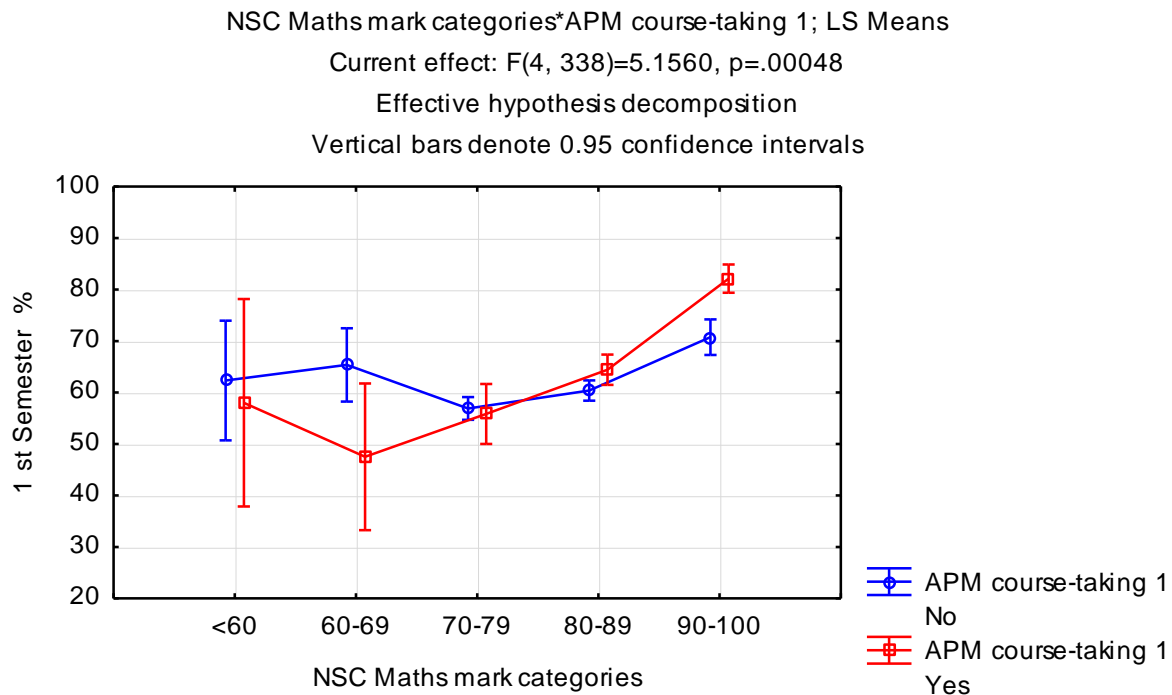


Figure 5.11: Difference between groups in first-semester Mathematics across NSC Mathematics categories

The performance of the APM Yes group was significantly better than the performance of the APM No group in the first semester in the NCS Mathematics (Figure 5.11). From the graph it seems as if the APM No group performed better than the APM Yes group in the lower categories. This result should be interpreted acknowledging the fact that an average of 70% is the minimum requirement for enrolling in the first-year Mathematics courses. The sample size of the data in the categories below 70% is therefore very small (APM No group totals 11 out of 232, i.e. 4%, and APM Yes group 3 out of 116, i.e. 2%). The statistics on the performance in the APM Yes group over the lower categories

are therefore not relevant for this study and are merely included for the sake of completeness.

(b) Second-semester Mathematics across NSC Mathematics categories

Table 5.12 presents the results of the two way Anova and Bonferroni test used to determine difference between the means of the two groups in the second-semester Mathematics per NSC Mathematics category. There is no significant difference, which means that the APM Yes and APM No groups do not differ between the different NSC categories.

Table 5.12: Anova and Bonferroni tests for difference in second-semester Mathematics performance across NSC Mathematics categories

			Univariate Tests of Significance for 2nd Semester % (DATA 201- Sigma-restricted parameterization Effective hypothesis decomposition				
Effect	SS	Degr. of Freedom	MS	F	p		
Intercept	104597.2	1	104597.2	870.3161	0.000000		
NSC Maths mark categories	13768.4	4	3442.1	28.6405	0.000000		
APM course-taking 1	352.8	1	352.8	2.9357	0.087680		
NSC Maths mark categories*APM course-taking 1	667.4	4	166.8	1.3883	0.237938		
Error	35934.7	299	120.2				

Bonferroni test; variable 2nd Semester % (DATA 20140726 in Grafieke 20140826.stw) Probabilities for Post Hoc Tests Error: Between MS = 120.18, df = 299.00												
Cell No.	NSC Maths mark categories	APM course-taking 1	{1} 53.625	{2} 53.667	{3} 58.763	{4} 58.860	{5} 54.500	{6} 45.000	{7} 70.812	{8} 73.815	{9} 52.500	{10} 27.000
1	70-79	No		1.000	0.175	0.721	1.000	1.000	0.000	0.000	1.000	0.745
2	70-79	Yes	1.000		1.000	1.000	1.000	1.000	0.002	0.000	1.000	0.977
3	80-89	No	0.175	1.000		1.000	1.000	1.000	0.000	0.000	1.000	0.190
4	80-89	Yes	0.721	1.000	1.000		1.000	1.000	0.000	0.000	1.000	0.196
5	60-69	No	1.000	1.000	1.000	1.000		1.000	0.042	0.002	1.000	0.940
6	60-69	Yes	1.000	1.000	1.000	1.000	1.000		0.945	0.435	1.000	1.000
7	90-100	No	0.000	0.002	0.000	0.000	0.042	0.945		1.000	1.000	0.005
8	90-100	Yes	0.000	0.000	0.000	0.000	0.002	0.435	1.000		0.330	0.001
9	<60	No	1.000	1.000	1.000	1.000	1.000	1.000	1.000	0.330		1.000
10	<60	Yes	0.745	0.977	0.190	0.196	0.940	1.000	0.005	0.001	1.000	

In Figure 5.12 below these results are shown graphically. For a specific NSC Mathematics category, there is no significant difference between the APM yes and no groups.

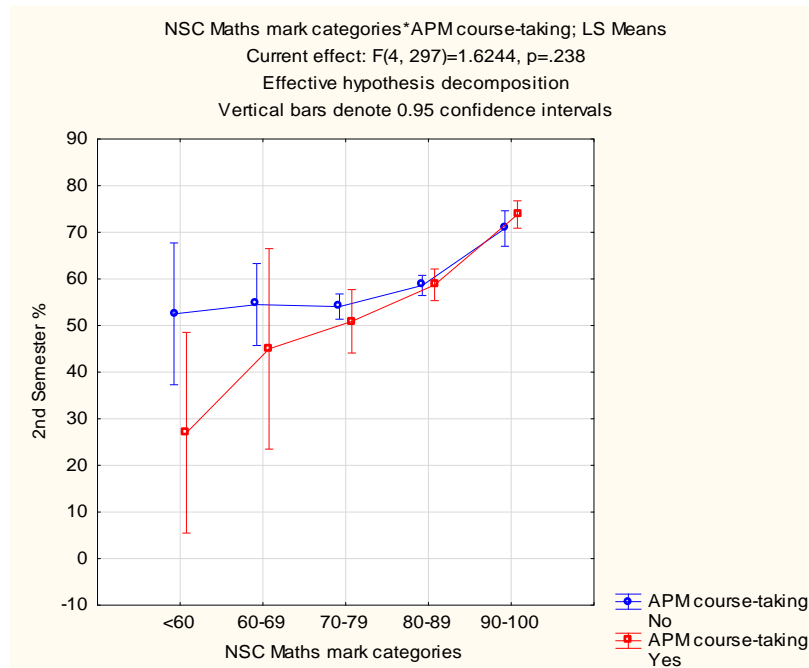


Figure 5.12: Difference between groups in second-semester Mathematics performance across NSC Mathematics categories

(c) First-semester Mathematics across NBT categories

Because of the uncertainties regarding the predictive value of the NSC Mathematics qualification as discussed in section 3.3.2(b), it is necessary to benchmark the results of the NSC Mathematics against another prior learning variable, i.e. the results of the NBT Mathematics.

The next analysis was done to determine the relationship between the NBT marks and the first-year Mathematics marks performance category. Table 5.13 shows the result of the Bonferroni test to determine on which means the APM Yes and APM No groups differed.

Table 5.13: Anova and Bonferroni tests for difference in first-semester Mathematics across NBT Mathematics categories

Bonferroni test; variable 1 st Semester % (DATA 20140726 in Grafieke 20140826.stw)												
Probabilities for Post Hoc Tests												
Error: Between MS = 118.06, df = 265.00												
Cell No.	NBT Maths categories	APM course-taking 1	{1} 56.471	{2} 58.737	{3} 60.095	{4} 62.875	{5} 64.963	{6} 72.750	{7} 68.417	{8} 83.815	{9} 82.000	{10} 89.571
1	<60	No		1.000	1.000	0.510	0.021	0.000	0.019	0.000	0.000	0.000
2	<60	Yes	1.000		1.000	1.000	1.000	0.001	0.737	0.000	0.006	0.000
3	60-69	No	1.000	1.000		1.000	1.000	0.000	0.902	0.000	0.007	0.000
4	60-69	Yes	0.510	1.000	1.000		1.000	0.055	1.000	0.000	0.057	0.000
5	70-79	No	0.021	1.000	1.000	1.000		0.376	1.000	0.000	0.168	0.000
6	70-79	Yes	0.000	0.001	0.000	0.055	0.376		1.000	0.009	1.000	0.014
7	80-89	No	0.019	0.737	0.902	1.000	1.000	1.000		0.003	1.000	0.003
8	80-89	Yes	0.000	0.000	0.000	0.000	0.000	0.009	0.003		1.000	1.000
9	90-100	No	0.000	0.006	0.007	0.057	0.168	1.000	1.000	1.000		1.000
10	90-100	Yes	0.000	0.000	0.000	0.000	0.000	0.014	0.003	1.000	1.000	

Univariate Tests of Significance for 1 st Semester % (DATA 20140726 in Grafieke 20140826.stw)					
Sigma-restricted parameterization					
Effective hypothesis decomposition					
Effect	SS	Degr. of Freedom	MS	F	p
Intercept	683900.5	1	683900.5	5792.799	0.000000
NBT Maths categories	13411.7	4	3352.9	28.400	0.000000
APM course-taking 1	1790.6	1	1790.6	15.167	0.000125
NBT Maths categories*APM course-taking 1	1158.8	4	289.7	2.454	0.046288
Error	31286.0	265	118.1		

As seen in Table 5.13, there is a significant interaction ($p=0.046$) between the means of the APM Yes and APM No groups in their first-semester Mathematics marks across the NBT Mathematics categories. This means that the difference between the APM Yes and APM No groups are not the same across the different categories of the NBT Mathematics marks. There is only a significant difference between the means of the two groups in the 80-89% category. For all the other NBT categories the Bonferroni test results show that the APM Yes group did not perform significantly better than the APM No group in first-semester Mathematics over the different NBT categories. This can be seen graphically in Figure 5.13.

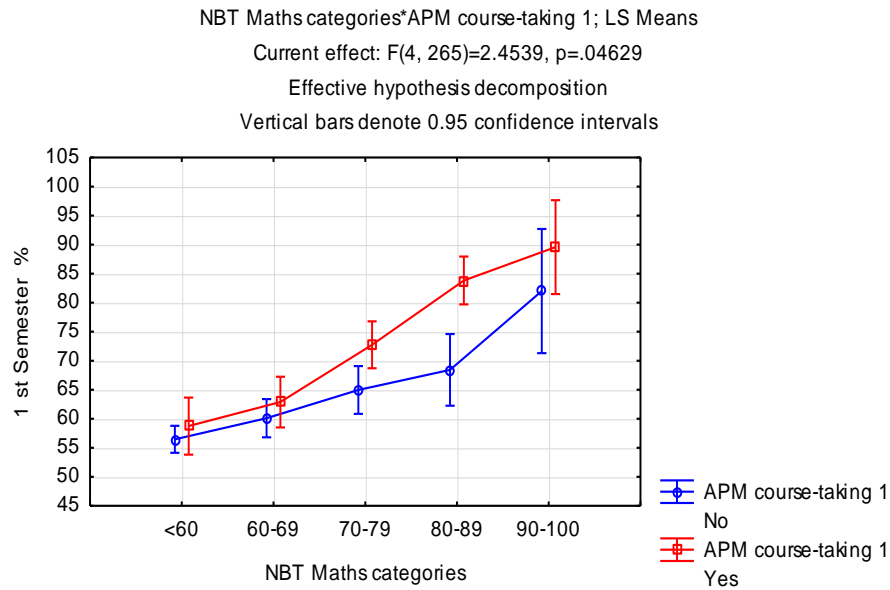


Figure 5.13: Difference between two groups in first-semester Mathematics performance across NBT Mathematics categories

(d) Second-semester Mathematics across NBT Mathematics

Table 5.14 and Figure 5.14 show the difference between the two groups APM Yes and APM No across NBT mark categories in the second-semester Mathematics. It seems that APM course-taking had a smaller influence on the results in the second semester of the first-year Mathematics than in the first-semester.

Table 5.14: Anova and Bonferroni tests for difference in second-semester Mathematics across NBT Mathematics categories

Bonferroni test; variable 2nd Semester % (DATA 20140726 in Grafieke 20140826.stw)												
Probabilities for Post Hoc Tests												
Error: Between MS = 142.15, df = 247.00												
Cell No.	NBT Maths categories	APM course-taking 1	{1} 57.156	{2} 57.188	{3} 60.425	{4} 57.333	{5} 63.040	{6} 65.143	{7} 68.000	{8} 76.926	{9} 78.250	{10} 80.000
1	<60	No		1.0000	1.0000	1.0000	1.0000	0.1196	0.1665	0.0000	0.0297	0.0001
2	<60	Yes	1.0000		1.0000	1.0000	1.0000	1.0000	0.8245	0.0000	0.0798	0.0015
3	60-69	No	1.0000	1.0000		1.0000	1.0000	1.0000	1.0000	0.0000	0.2127	0.0037
4	60-69	Yes	1.0000	1.0000	1.0000		1.0000	1.0000	0.6344	0.0000	0.0663	0.0009
5	70-79	No	1.0000	1.0000	1.0000	1.0000		1.0000	1.0000	0.0017	0.8373	0.0456
6	70-79	Yes	0.1196	1.0000	1.0000	1.0000	1.0000		1.0000	0.0137	1.0000	0.1573
7	80-89	No	0.1665	0.8245	1.0000	0.6344	1.0000	1.0000		1.0000	1.0000	1.0000
8	80-89	Yes	0.0000	0.0000	0.0000	0.0000	0.0017	0.0137	1.0000		1.0000	1.0000
9	90-100	No	0.0297	0.0798	0.2127	0.0663	0.8373	1.0000	1.0000	1.0000		1.0000
10	90-100	Yes	0.0001	0.0015	0.0037	0.0009	0.0456	0.1573	1.0000	1.0000	1.0000	

Univariate Tests of Significance for 2nd Semester % (DATA 20140726 in Grafieke 20140826.stw)					
Sigma-restricted parameterization					
Effective hypothesis decomposition					
Effect	SS	Degr. of Freedom	MS	F	p
Intercept	597220.1	1	597220.1	4201.463	0.000000
NBT Maths categories	8421.5	4	2105.4	14.811	0.000000
APM course-taking 1	128.2	1	128.2	0.902	0.343293
NBT Maths categories*APM course-taking 1	779.0	4	194.7	1.370	0.244874
Error	35110.0	247	142.1		

NBT Maths categories*APM course-taking 1; LS Means

Current effect: $F(4, 247)=1.3700$, $p=.24487$

Effective hypothesis decomposition

Vertical bars denote 0.95 confidence intervals

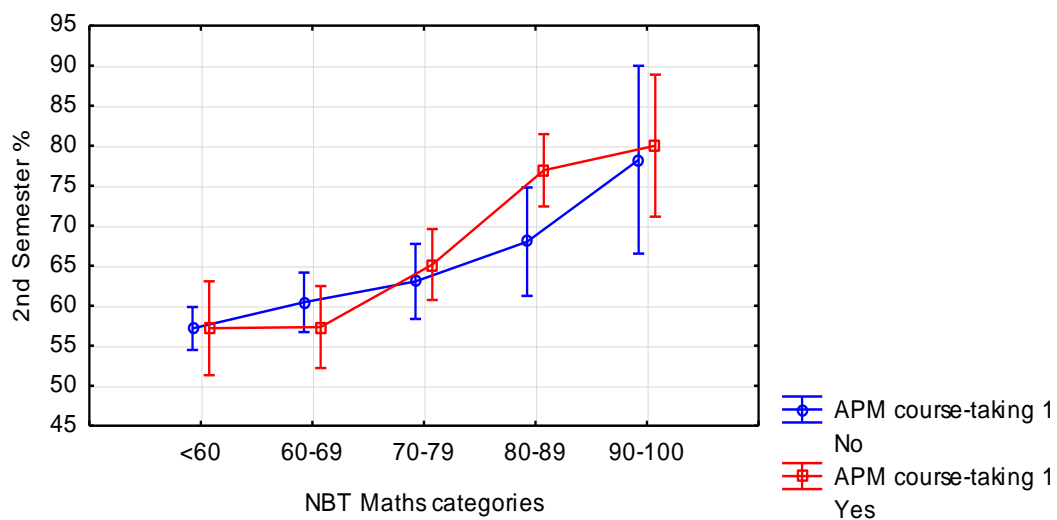


Figure 5.14: Difference between two groups in second-semester Mathematics performance across NBT Mathematics categories

Figure 5.14 shows that the means of the APM Yes group are still higher than those in the APM No group in the second semester in the NBT Mathematics categories over the categories 70-79%, 80-89% and 90-100%, although the difference is not significant ($p=0.2$).

5.3.3 Inferential statistics of Sample A

(a) Relationship between prior learning results and first-year university Mathematics

The prior learning results and the first-year Mathematics results can be assumed to be continuous variables and compared as such using regression and correlation analysis. It is necessary to determine if the influence that the independent variable (IV) has over the response or dependent variable (DV) is significant or not. Assuming a straight-line regression of the IV over the response variable, one has to determine the slope of the regression line. If the slope is so close to zero that one cannot distinguish it from zero, then the IV has no influence on the DV and one states that they are unrelated. The r -value gives the correlation coefficient and the r^2 value is the coefficient of determination that reveals the proportion of changes in DV explained by changes in IV (Taylor, 1990).

In an analysis of the relationships between the prior learning variables and first-year university Mathematics, the following results shown in Table 5.15 are obtained:

Table 5.15: Results of correlation analysis

Independent Variable	Dependent Variable		N	r-value	p-value	r^2 value (coefficient of determination)
NSC Mathematics	First-semester Mathematics	Group	1199	0.53	0.0000	0.28
		APM Yes	152	0.73	0.0000	0.53
		APM No	260	0.30	0.0000	0.08
NBT Mathematics	First-semester Mathematics	Group	1199	0.57	0.0000	0.32
		APM Yes	141	0.70	0.0000	0.32
		APM No	192	0.42	0.0000	0.18

Independent Variable	Dependent Variable		N	r-value	p-value	r ² value (coefficient of determination)
GR 12 average	First-semester Mathematics	Group	1199	0.47	0.0000	0.22
		APM Yes	159	0.63	0.0000	0.39
		APM No	181	0.33	0.0000	0.11
NSC Paper 3	First-semester Mathematics	Group	1199	0.54	0.0000	0,29
		APM Yes	109	0.60	0.0000	0.37
		APM No	119	0.57	0.0000	0.32
NSC Mathematics	Second-semester Mathematics	Group	1199	0.53	0.0000	0.23
		APM Yes	152	0.66	0.0000	0.44
		APM N	260	0.45	0.0000	0.20
NBT Mathematics	Second-semester Mathematics	Group	1199	0.44	0.0000	0.18
		APM Yes	141	0.57	0.0000	0.32
		APM No	192	0.38	0.0000	0.15
GR 12 average	Second - semester Mathematics	Group	1199	0.47	0.0000	0.22
		APM YES	159	0.56	0.0000	0.31
		APM NO	181	0.53	0.0000	0.27
NSC Paper 3	Second-semester Mathematics	GROUP	1199	0.43	0.0000	0.19
		APM YES	109	0.50	0.0000	0.24
		APM NO	119	0.21	0.03	0.05

Based on the analysis it is clear that in each of the variables in Table 5.15 above, the IV contributed significantly to the DV. This means there is a positive correlation between all the IVs and the DVs, indicating that if a student performed well in the independent prior learning variable in either the target group or the APM Yes or APM No group, he or she would probably also perform well in the first-year Mathematics examinations. The correlations between the NSC Mathematics marks and the NBT Mathematics marks of the APM Yes students and the first semester mark ($r=0.73$ and $r=0.70$ respectively) were the strongest of all the correlations. This indicates that these two marks would

probably predict better performance in the first semester examination than the Grade 12 average mark or the NSC Paper 3 mark. In all four variables summarised in Table 5.15, the correlation with the APM Yes group was stronger than the correlation with the APM No group. Because of the small sample size in NSC Paper 3, this variable was excluded in any further analyses.

(b) Combination of all variables in a general regression model

The final step in the statistical analyses was to determine the influence of the different variables on the performance in first-semester Mathematics when the variables are combined. To be able to do this, a general regression model is fit to the available data. The predictor variables that are combined for the analysis on first semester performance are the NSC Mathematics marks, the APM marks and the NBT marks. All these variables had positive parameters, which mean that the higher the category was, the better the marks in first-semester Mathematics were. The results of this analysis are shown in Tables 5.16 and 5.17:

Table 5.16: General regression model on first semester performance

Effect	Parameter Estimates (DATA 20140731) Sigma-restricted parameterization						
	1 st Semester % Param.	1 st Semester % Std.Err	1 st Semester % t	1 st Semester % p	-95.00% Cnf.Lmt	+95.00% Cnf.Lmt	1 st Semester % Beta (β)
Intercept	-882.561	186.4408	-4.73373	0.000012	-1254.80	-510.320	
NBT Maths categories	4.194	0.9916	4.22925	0.000074	2.21	6.173	0.414588
NSC Maths mark categories	4.857	1.9902	2.44034	0.017366	0.88	8.830	0.235783
APM Gr 12 Mark categories	3.138	0.9862	3.18172	0.002232	1.17	5.107	0.299528

Table 5.17: Test of whole model on first semester performance

Dependent Variable	Test of SS Whole Model vs. SS Residual (DATA 20140731)										
	Multiple R	Multiple R ²	Adjusted R ²	SS Model	df Model	MS Model	SS Residual	df Residual	MS Residual	F	p
1 st Semester %	0.82724	0.68434	0.66999	10158.8	3	3386.29	4685.91	66	70.99861	47.6952	0.00000

The beta coefficient (as seen in Table 5.16) reflects the relative impact of each of the independent variables on the dependent variable and in this case the largest positive impact in the prediction of first semester performance is the NBT Mathematics mark of a student ($\beta=0.414$). The second best predictor is the APM mark ($\beta=0.3$) and the variable with the smallest influence on the first semester marks is the NSC Mathematics mark

($\beta=0.23$). The coefficient of determination ($R^2=0.68$) shows that the IVs in this model explain 68% of variance in the first-semester Mathematics marks.

This finding that both the NBT Mathematics Marks and the APM marks are better predictors of success in first-semester Mathematics is a new contribution in the knowledge domain of predictors of early undergraduate Mathematics success. To the researcher's knowledge, no analyses have yet been done on the predictive value of APM marks. The relative newness of the NSC and NBT implies that little research has been published on their signalling abilities (Rankin, Schöer; Sebastiao & Van Walbeek, 2012). However, the literature still suggests that on average NSC remains a reasonable predictor of university success (Hunt, Ntuli, Rankin, Schöer & Sebastiao, 2011; Nel & Kistner, 2009). According to these studies, criticism about the NSC stems from the fact that the lower performance ranges have inflated marks. Once deflated, the NSC remains a reasonable predictor of university success. According to Potgieter, Davidowitz and Venter (2008) prediction of success is typically more accurate for better performing students.

This finding on the predictive value of APM is in accordance to the claims of Klopfenstein and Thomas (2010) that AP course-taking alone is predictive of university success, and the claims on the predictive value of AP –Calculus in STEM persistence by Ackerman, Kanfer and Beier (2013).

The results of the general regression model in the second semester are illustrated in Table 5.18 and 5.19.

Table 5.18: General regression model on second-semester performance

Effect	Parameter Estimates (DATA 20140731)							
	Sigma-restricted parameterization							
	2nd Semester % Param.	2nd Semester % Std.Err	2nd Semester % t	2nd Semester % p	-95.00% Cnf.Lmt	+95.00% Cnf.Lmt	2nd Semester % Beta (β)	2nd Semester % St.Err.β
Intercept	-635.682	244.4667	-2.60028	0.011591	-1124.21	-147.154		
NBT Maths categories	3.781	1.3178	2.86913	0.005595	1.15	6.415	0.343955	0.119881
NSC Maths mark categories	2.839	2.6021	1.09116	0.279357	-2.36	8.039	0.128466	0.117733
APM Gr 12 Mark categories	4.083	1.2711	3.21176	0.002079	1.54	6.623	0.368929	0.114868

Table 5.19: Test of whole model on second-semester performance

Dependent Variable	Test of SS Whole Model vs. SS Residual (DATA 20140731)										
	Multiple R	Multiple R ²	Adjusted R ²	SS Model	df Model	MS Model	SS Residual	df Residual	MS Residual	F	p
2nd Semester %	0.72830	0.53042	0.51575	8438.04	2	4219.02	7469.98	64	116.718	36.1470	0.00000

The results in Table 5.18 illustrate that NSC Mathematics has no significant correlation with second-semester performance ($p > 0.05$) and it is therefore excluded from the analysis. The correlation coefficients of NBT Mathematics ($\beta = 0.34$) and APM ($\beta = 0.37$) are significant. The APM mark is the strongest predictor of success in the second semester, although a weaker predictor than in the first-semester, since the correlation coefficient of the whole model is lower than in the first semester ($r = 0.7$).

In the final analysis (Table 5.20), the first semester mark was added to the IVs in the general regression model.

Table 5.20: General regression model on second-semester performance (with first-semester)

			Parameter Estimates (DATA 20140726 in Grafieke 20140826.stw)									
			Sigma-restricted parameterization									
Effect	Level of Effect	Column	2nd Semester % Param.	2nd Semester % Std.Err	2nd Semester % t	2nd Semester % p	-95.00% Cnf.Lmt	+95.00% Cnf.Lmt	2nd Semester % Beta (β)	2nd Semester % St.Err.β	-95.00% Cnf.Lmt	+95.00% Cnf.Lmt
Intercept		1	-3.388	13.248	-0.256	0.801	-31.339	24.563				
1 st Semester %		2	0.982	0.193	5.091	0.000	0.575	1.389	0.925	0.182	0.542	1.309
NBT Maths categories	60-69	3	0.626	3.380	0.185	0.855	-6.506	7.758	0.022	0.118	-0.227	0.271
NBT Maths categories	<60	4	2.830	4.818	0.587	0.565	-7.335	12.996	0.080	0.137	-0.208	0.368
NBT Maths categories	70-79	5	3.043	3.842	0.792	0.435	-5.064	11.149	0.092	0.116	-0.152	0.336
NBT Maths categories	80-89	6	-3.218	3.212	-1.002	0.330	-9.995	3.559	-0.106	0.106	-0.329	0.117
NSC Maths mark categories	70-79	7	-2.444	5.735	-0.426	0.675	-14.544	9.657	-0.100	0.236	-0.598	0.397
NSC Maths mark categories	80-89	8	0.704	3.496	0.201	0.843	-6.671	8.079	0.043	0.213	-0.407	0.493
NSC Maths mark categories	60-69	9	2.227	7.002	0.318	0.754	-12.547	17.001	0.083	0.261	-0.468	0.634
APM Gr 12 Mark categories	1	10	-0.866	4.017	-0.216	0.832	-9.342	7.610	-0.034	0.155	-0.370	0.302
APM Gr 12 Mark categories	2	11	1.119	3.675	0.304	0.765	-6.634	8.871	0.043	0.140	-0.253	0.338
APM Gr 12 Mark categories	3	12	-2.653	2.805	-0.946	0.358	-8.571	3.265	-0.109	0.115	-0.352	0.134
APM Gr 12 Mark categories	4	13	-2.308	3.663	-0.630	0.537	-10.036	5.421	-0.084	0.133	-0.364	0.197

It is of interest to note that the first semester mark outperforms the NSC mark ($p < 0.001$), the NBT mark as well as the APM mark in the prediction of the second-

semester Mathematics mark. The first semester mark is therefore the best predictor of success in second-semester Mathematics.

As indicated in section 5.2, it is difficult to compare the prior learning Mathematics curricula with the first year Mathematics curricula as there are too many cognitive and pedagogical differences. It is therefore obvious that the first-semester Mathematics marks would be the best predictor of second semester success when compared to the prior learning results. Du Preez, Steyn and Owen (2008) confirm this finding.

The important part of this finding is that the APM marks outperformed the NSC Mathematics marks as a predictor of second semester success, even though its predictive value was much weaker than in the first-semester.

5.3.4 Summary of Section 5.3

This section attempted to answer the following secondary research question: *To what extent, if any, did the students who took the APM courses in high school and wrote the final examination perform better in the first-year Mathematics examinations than those who did not take the course?*

The analyses of the data showed that the students who had taken APM performed better in first-semester Mathematics than their peers, but APM course-taking had no significant effect on the second-semester mark. The APM marks of students proved to be a better predictor of success in first-semester Mathematics as well as second-semester Mathematics than the NSC Mathematics marks. When all the variables were combined, the best predictor of success in the second semester was the first-semester Mathematics mark.

5.4 SECONDARY RESEARCH QUESTION C

This question reads as follows:

What are the student's perceptions on, and experience of, the effectiveness of APM in easing the transition between school and university Mathematics?

This section will determine the perceptions of the students on the usefulness of APM, by analysing the data obtained from the questionnaire. The aim was to reveal important and meaningful patterns and relationships contained in the data. The sample group consisted of all the students who completed the questionnaire (n=59). Of these students, 49 students in the sample group, i.e. 30.1%, indicated that they had taken APM. To avoid confusion with the previous sample, this sample group will be referred to as Sample group B.

To get a better idea of the profile of the students who completed the questionnaire (Sample B), questions 1, 2, 3 and 10 will be analysed first.

5.4.1 Profile of students in Sample B

The first three questions in the questionnaire were aimed at all the students who answered the questionnaire.

Question 1 asked whether a student was repeating either a first- or second-semester Mathematics course or both. Figure 5.15 shows the result:

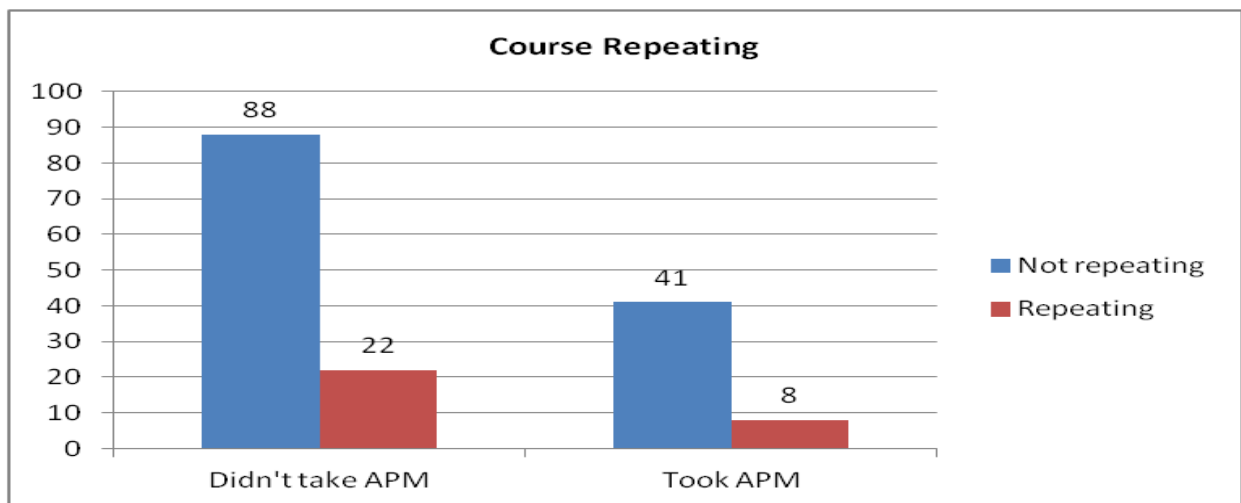


Figure 5.15: Number of students repeating a Mathematics course

Eight, (16%) of the students who had taken APM in Grade 12 (APM Yes students), were repeating a semester course and twenty two, (20%) of the students who had not taken APM (the APM No students), were repeating a course. Of the APM Yes students, two

were repeating a first semester course, one was repeating a second semester course and five were repeating both courses.

Question 2 asked the students to rate the level of difficulty they experienced in the transition from school to university Mathematics on an ordinal scale from 1 to 5, where 1 represented a high level of difficulty and 5 a low level of difficulty. Figure 5.16 shows the result:

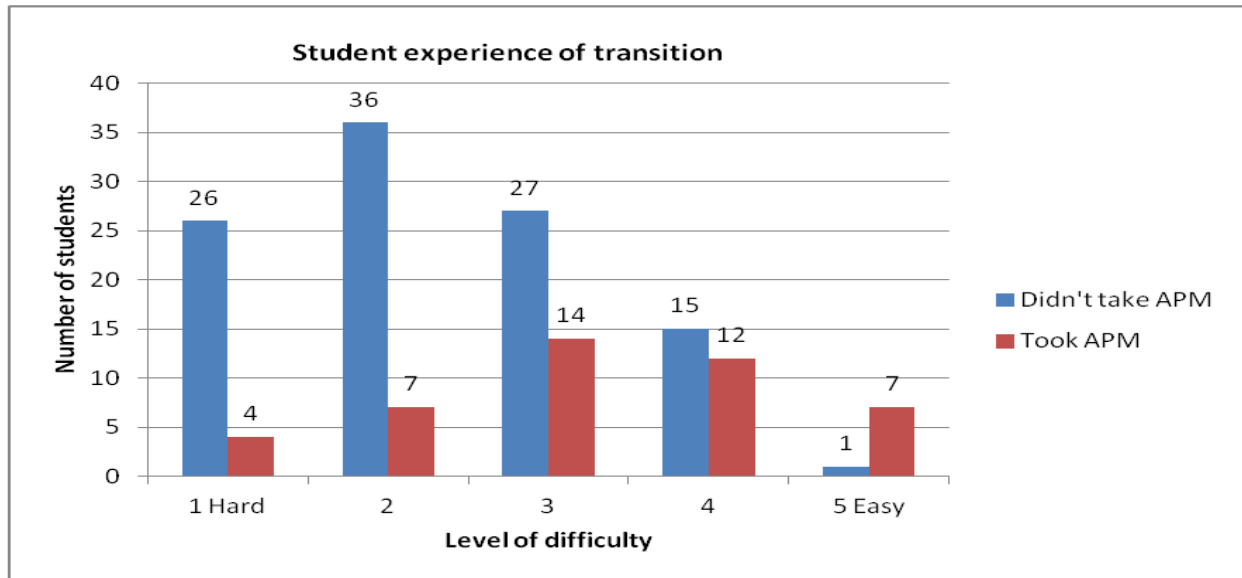


Figure 5.16: Level of difficulty experienced in the transition from school to university Mathematics

The average level of difficulty the students experienced, i.e. the mean of the APM Yes group, was 3.25 (65%), while the mean of the APM No group was 2.23 (46%). This implies that there is a relationship between APM course-taking and the level of difficulty experienced in the transition from school to university. The degree of significance of this relationship will be statistically verified in section 5.4.2(a).

Question 3 asked the students to tick the mark category which best described their NSC mark for Mathematics. Figure 5.17 shows the distribution of the NSC Mathematics marks:

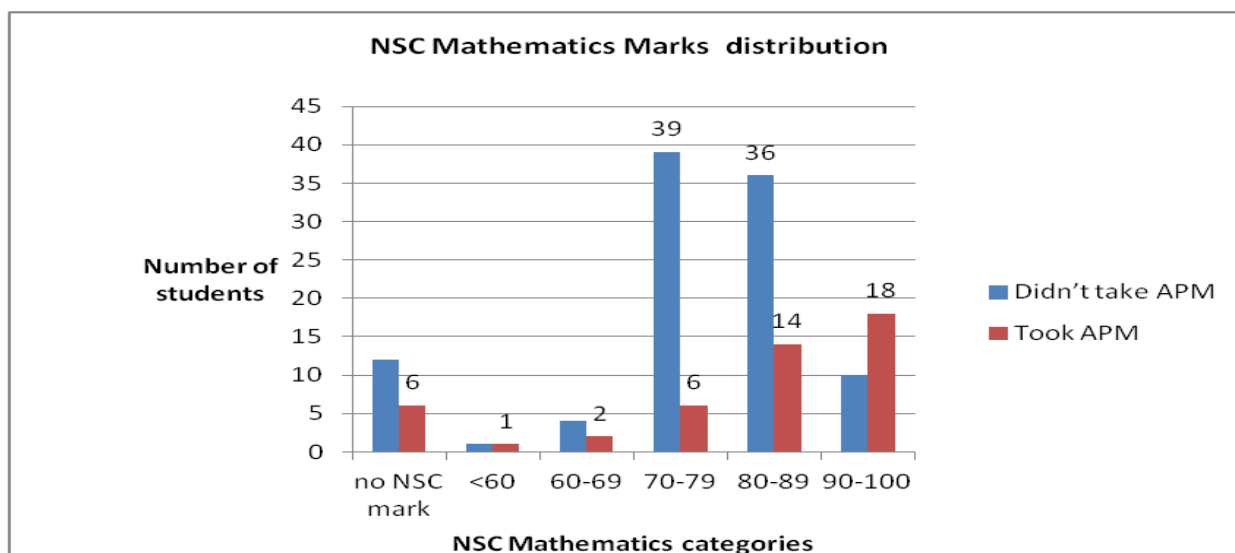


Figure 5.17: Distribution of the NSC Mathematics marks for Sample B

Questions 4 - 10 were only applicable to students who had taken the APM course.

Question 9 asked the APM students to rate the mathematical knowledge of their APM teacher. The mean percentage of these ratings was 66%.

Question 10 asked the students in which type of high school they had matriculated. Figure 5.18 summarises the result:

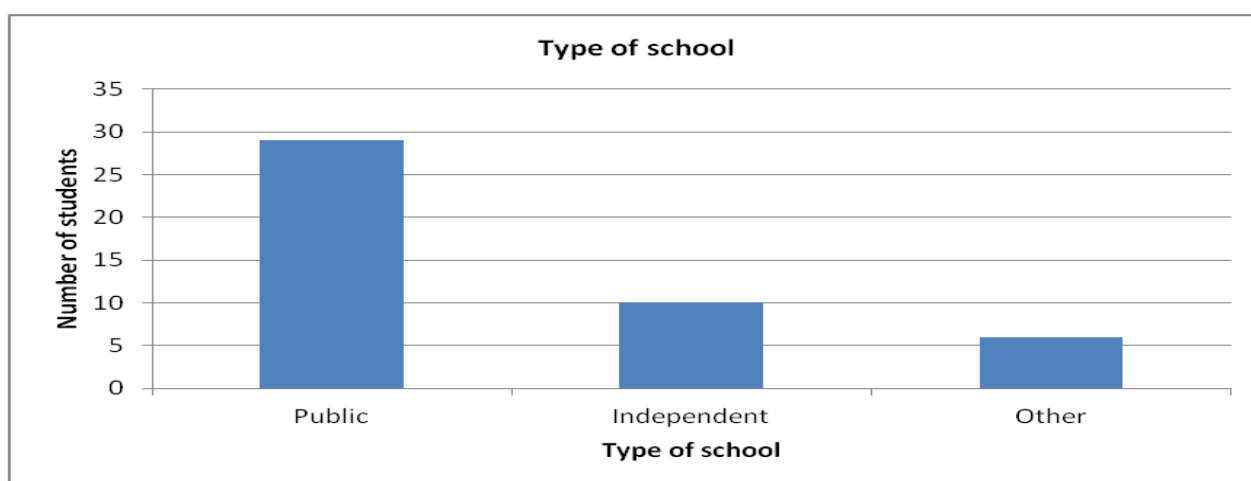


Figure 5.18 Type of school attended

5.4.2 Perception of the students in Sample B of the transition from school Mathematics to university Mathematics

A chi-square test was done to determine the degree of relationship between APM course-taking and students' perception of the level of difficulty of the transition process between school Mathematics and university Mathematics. When the students were asked to rate their experience on a scale from 1 to 5, where 1 represented a very hard experience, the mean of the experience of the APM Yes group was 3.25 (65%) and the mean of the APM No group was 2.23 (46%). The chi-square ($df=4$) =23.3 and $p<0.01$, and therefore there is a significant relationship between APM course-taking and the level of difficulty experienced by students.

5.4.3 Students' perceptions of the effect of APM course-taking

(a) Time-management skills

Question 5b asked students to indicate the effect of APM-course-taking on their time-management skills on a scale of 1 to 5. A choice of 1 indicated that APM course-taking did not help them to acquire time-management skills, and a choice of 5 indicated that APM course-taking did help them to acquire time-management skills. The mean of the student's response was 3.4 or 68%, the median 4 (80%) and the standard deviation 1.2. Because the DV is ordinal, the median value (median =4 or 80%) would be the best indicator of the 'most typical case'. This median value shows that the average student experienced that APM Mathematics course-taking improved time-management skills.

To test the degree and significance of the relationship between the different APM mark categories and time-management skills, which are both ordinal variables, a Spearman rank correlation analysis ($r=0.34$ and $p=0.02$) was performed. These values show that there is a moderately positive correlation between the variables APM course-taking and time-management skills, but that no causality is implied.

A correlation analysis between the students' reported ability to manage their time and their first semester marks produced an r value of 0.23 and $p=0.07$. This shows that there is no significant interaction between these two variables.

(b) Skills in handling the large workload

Question 5c asked students to indicate if APM-course-taking taught them to handle a large workload, on a scale of 1 to 5. A choice of 1 indicated that APM course-taking did not teach them the skill to handle a large workload and a choice of 5 indicated that APM course-taking did teach them the skill. The mean of the students' response was 3.9 (78%). To test the degree and significance of the relationship between the different APM mark categories and the skill to handle a large workload, which are both ordinal variables, a Spearman rank correlation analysis ($r=0.3$ and $p=0.04$) was done. These values show that there was a moderate positive correlation between the variables APM course-taking and the skill to handle large a workload, but that no causality was implied.

A correlation analysis between the APM course-taking students' reported ability to handle a large workload and their first-semester Mathematics marks produced a r -value of 0.18 and a p -value of 0.23. There is no significant interaction between the two variables.

(c) Understanding Mathematics

Question 5d asked students to rate the effect of their APM course-taking on their overall understanding of Mathematics on a scale of 1 to 5. The choice of 1 indicated that APM course-taking did not help them to understand Mathematics better at all, and a choice of 5 indicated that APM course-taking did the opposite. The median of the students' response was 4 (80%), which meant that the average student definitely experienced that APM course-taking did help him or her to understand Mathematics better.

To test the degree and significance of the relationship between the different APM mark categories and a student's understanding of Mathematics, a Spearman rank correlation analysis ($r=0.05$ and $p=0.72$) was done. These values show that there is no significant relationship between the variables APM course-taking mark categories and understanding of Mathematics.

In the correlation analysis between the students' perceived understanding of Mathematics and first-semester Mathematics marks, $r=0.28$ and $p=0.0501$. This is a

borderline case and shows that there is no significant interaction between how students perceived their understanding of Mathematics and their first-semester marks.

(d) Confidence building

Question 5c asked students to indicate if APM-course-taking improved their confidence in their mathematical abilities. On a scale from 1 to 5, the choice of 1 indicated that it definitely did not, and a choice of 5 indicated it definitely did improve their confidence. The mean of the student's response was 5 (100%) and the standard deviation was 0.83. This means the average student experienced that APM helped him or her to have more confidence in his or her own mathematical abilities.

To test the degree and significance of the relationship between the different APM mark categories and confidence, which are both ordinal variables, a Spearman rank correlation analysis ($r=0.34$ and $p=0.02$) was done. These values show that there is a moderate positive correlation between the variables APM course-taking and confidence boosting, but that no causality is implied.

A correlation analysis between the students' perceived confidence and their first semester marks produced an r value of 0.38 and $p=0.00$, which shows significant interaction between the two variables. For the second semester it was $r=0.30$ and $p=0.049$. This result also shows a significant relationship.

Figures 5.19 and 5.20 show the regression line between confidence and university Mathematics marks:

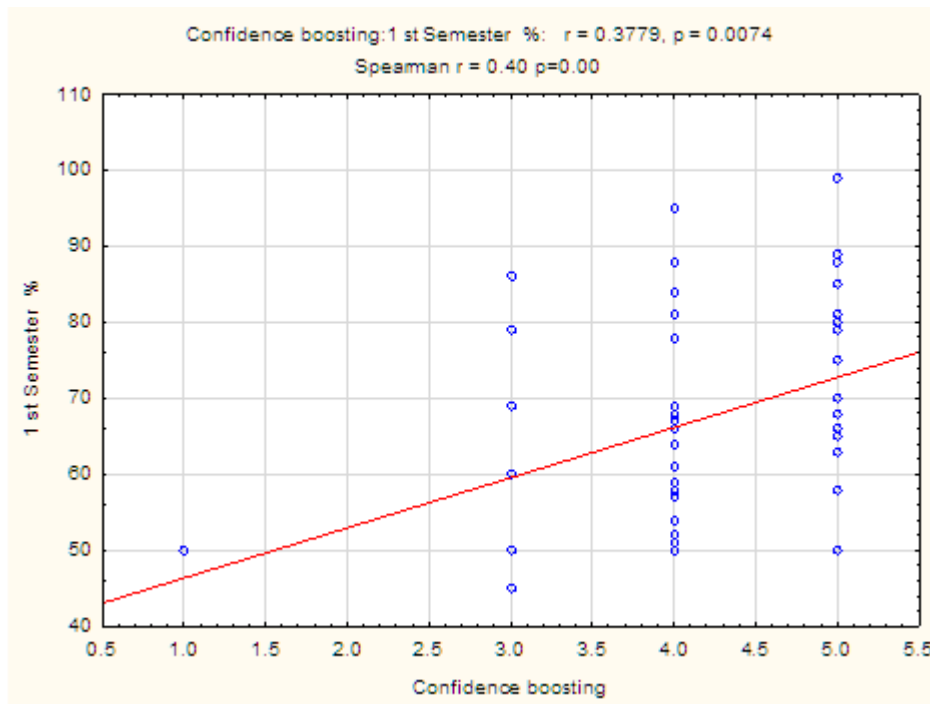


Figure 5.19: The relationship between confidence and first-semester Mathematics marks

The strong positive relationship between the confidence level of the student and performance in the first-semester Mathematics examination, as seen in Figure 5.19, correlates with the claim of Stankov (2014) that confidence is a very good predictor of success in Mathematics. It also supports the view of Ernest (2000) that growth in confidence means epistemological and mathematical empowerment.

The correlation between perceived confidence and performance in the second-semester Mathematics is still significant although not as strong as in the first semester as seen in Figure 5. 20.

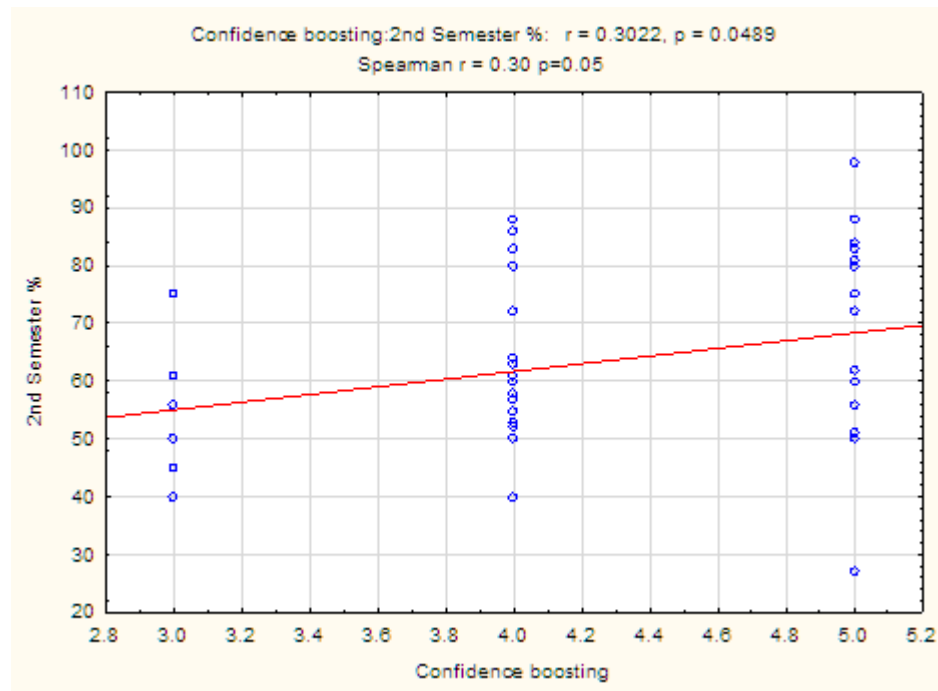


Figure 5.20: The relationship between confidence and second-semester Mathematics marks

5.4.4 Summary of Section 5.4

The questionnaire asked the students to rate their experience of five virtues of APM course-taking. The results showed a significant relationship between APM course-taking and the level of difficulty experienced by students in the transition process between school Mathematics and university Mathematics. The results showed no significant interaction correlation between APM maths course-taking ($p=0.23$) and the improvement of time-management skills and the skill to handle a large workload ($p=0.04$).

It is interesting to note that there was no significant correlation between a student's perception of his comprehension of Mathematics and his/her APM or NSC Mathematics marks. However, there was a significant interaction between the student's perceived confidence in Mathematics and first semester marks ($p=0.0074$) as well as second semester marks.

APM course-taking definitely eases the initial transition from school to university in the sense that the students feel that they ‘understand’ the Mathematics when they enter university. This gives them confidence and this confidence leads to an easier transition from school to university Mathematics. It definitely also influences their first-semester Mathematics marks positively as seen in section 5.3. This is in line with the work of Ackerman, Kanfer and Beier (2013) and Stankov (2014) who argue that students who have confidence in their mathematical ability and who devote the necessary effort to acquire domain knowledge in high school are the students that succeed in AP examinations and later in post-secondary study (see section 3.7).

5.5 VOICES OF THE STUDENTS IN THE OPEN-ENDED QUESTION

Question 7 in the questionnaire asked the following open-ended or unstructured question: *In which other ways did the fact that you took APM in high school have a positive or negative influence on your experience of first-year Mathematics?*

Thirty-five students answered this question and all their answers were analysed by the process of coding. This meant organising the answers into categories on the basis of themes, concepts and similar features. The researcher searched for patterns in the data, and these patterns were then interpreted in terms of the social theories stated in Chapter 3. “A good thematic code is one that captures the richness of the phenomenon. It is useable in the analysis, the interpretation and the presentation of research” (Boyatzis, 1998, p. 31). Plowright (2011) argues that an open question allows the participant a greater choice about what to write and they have more control over the information they will disclose. The researcher will have more freedom in how the data is interpreted, because the participants will not be restricted by having to use predetermined categories constructed before the data was collected.

The seven basic themes that stood out in the students’ answers were comprehension, enjoyment and enrichment of Mathematics, work tempo, skills, Mathematics anxiety, motivational factors and confidence. Each of these themes will be discussed separately in the next few paragraphs.

5.5.1 Comprehension

Because of the similarity of the syllabi of first-semester Mathematics and APM (see 5.1), the content of the university course was familiar to the students, and they could spend more time in trying to understand the concepts instead of just following algorithms or using imitative reasoning as is often seen in first year Mathematics courses at university (See section 3.3.3(c)). “APM helped me understand the basic concepts of first-year Mathematics”. This theme was echoed by several students. “It helped me not to learn Mathematics like a parrot anymore”. APM course-taking gave the students a good basis on which to build in their studies of university Mathematics.

APM students had already been exposed to some of the difficult topics such as integration and logarithmic functions, and therefore did not find it so overwhelming when it was introduced in the first-semester course. They could easily keep up in these topics and could even help others who struggled. One student mentioned that the integration he had already done in APM specifically helped him in the difficult subject Applied Mathematics B124. “Taking APM at school gave me insight into the Mathematics to come at university, and lightened the workload needed to fully grasp the work being covered at university”.

5.5.2 Enjoyment of Mathematics

Enjoyment of Mathematics was evident in many of the answers. Because the students were studying the same content for a second time, they already knew the basics and could spend more time attempting the challenging problems and experiencing the beauty of Mathematics. A few students also noted that APM at school level was much more interesting than the NSC Mathematics and that it was good enrichment for anyone interested in Mathematics. A student wrote: “Because the foundations were already built, I could truly enjoy the Mathematics!” Another wrote: “It gave me the spark to go study Engineering”.

One of the students mentioned that APM course-taking made university Mathematics extremely boring at first, since most of the work had already been covered. This was the only negative comment on APM but noteworthy. University lecturers in particular should

take note that they should also provide challenges for students in their first year classes. A student who had dropped APM because he thought it was a waste of time, could only see the relevance and challenge of APM once he was studying first-year Mathematics. Another student wrote: “APM encouraged me to go study further in Mathematics – it wasn’t easy, but it was worth trying!”

5.5.3 *Work tempo*

A few students mentioned that because APM was only taught for one period a week in most schools, they were already used to dealing with a large volume of work in one lesson. “I never felt overwhelmed by the volumes of work covered at university; in fact I felt that we were moving too slowly”. It helped students to understand the work faster and better because they were used to the fast work tempo in APM. “Having had APM in school allowed me more time to get used to university life in general”. It gave students the time to focus on other first semester subjects. One student warned that APM took a lot of time and that taking the course without spending time on it would have no benefits in the first year.

5.5.4 *Skills*

APM teaches students how to study independently. This is even relevant for those who do the online course. Several students felt that APM also improved their problem-solving skills, which are “crucial” for many STEM careers. Some students specifically mentioned that APM helped them to stay motivated to study for a long time. “APM is good preparation for any university subject because it teaches you how to work and how to manage a busy life. It teaches you to go home and try the difficult stuff again, and to make sure that you understand it. It teaches you how to write an examination against time, it teaches you to battle with questions and not to give up”. The students also compared APM to school Mathematics: “APM teaches you skills that normal school Mathematics never does, APM should be a school subject – introduced as a third level of Mathematics with each school having their own teacher”.

5.5.5 Mathematics anxiety

Because APM students were familiar with the content of the first year Mathematics course, and had learned to handle large workloads effectively in little time, they experienced much less mathematical anxiety than their peers. They knew that they were familiar with the content and had more time at hand. “Because I took APM in school, I had more time and I could adapt to university life with less stress. It made first-year Mathematics so much easier and I stressed much less than other students”.

“APM taught me to handle stress in a Mathematics exam and to work against time. I did not have to stress such a lot for a university examination in Mathematics because I knew that I could use my APM knowledge to tackle any problem”.

5.5.6 Confidence

The basic theme that underpinned all the above-mentioned responses was confidence: “APM gave me confidence in my mathematical abilities.” Students felt that having been exposed to the content of the first semester gave them a good basis from which to work, it helped them to work faster, to experience less anxiety and to know that they had the necessary problem-solving skills. “I knew that I could fall back on my APM knowledge”. In other words, having taken APM made them confident.

5.5.7 Summary of Section 5.5

In this section the perceptions of the students as reflected in the open question of the questionnaire were addressed. With the exception of only one negative comment, all the comments of the students were positive about the influence of APM course- and examination-taking. The students who took APM had already been exposed to most of the topics in first-year Mathematics. This made their workload easier and helped them to gain a better understanding of the basic concepts of Mathematics and also led to enjoyment of Mathematics.

APM also taught the students to handle large workloads in little available time, it taught them independent study skills, gave them experience in handling test anxiety and gave them confidence in their mathematical abilities. All of these new skills led to more time

to adapt to the challenges of student life in the first year. APM course-taking taught them a new way of thinking and learning.

5.6 SUMMARY AND INTEGRATION OF RESEARCH RESULTS

The research was guided by the following three secondary research questions:

- *How are the APM and NSC (Paper 1, 2 & 3) and NBT Mathematics curriculum related to the first-year Mathematics curriculum (at SU)?*
- *To what extent, if any, do the learners who take the APM course and examinations prior to admission to first year, perform better in their first-year university examinations in Mathematics?*
- *What are the students' perceptions of their experience of the effectiveness of AMP in easing the transition between school and university Mathematics?*

When integrating the results of the document analysis, the statistical data analyses and the interpretation of the voice of the students, it is clear that the students' 'APM experience' is in accordance with the conceptual framework of this study. In a hypothetical case, a high school learner with confidence in his mathematical skills enrolls for APM and works diligently from Grades 10 to 12. When writing the final APM examination, he will have acquired a good domain knowledge of Mathematics of which the content corresponds largely with that of first year university Mathematics (section 5.2.3). According to Bandura's self-efficacy theory the mastery experiences he experienced doing APM will have strengthened his belief in his self-worth and competence to do Mathematics (section 3.5.1).

Success in the APM examination will lead to confidence in his mathematical abilities and subsequent good marks in the NSC Mathematics examination. This most likely will allow access to a STEM course at a university. The student enters the first year Mathematics course with confidence which according to Stankov and Lee's (2014) 'gradient of predictability' model is the best predictor of any kind of cognitive performance (See section 3.5.2(b)). His confidence in Mathematics and good prior domain knowledge, when combined, lead to success in specifically the first semester of

his first-year Mathematics according to Ackerman, Kanfer and Beier (2013). Success in first semester Mathematics is a predictor of success in the second-semester Mathematics (section 5.3.3(b)). Success in the first year at university leads to self-confidence, a sense of mathematical self-efficacy and personal ownership of and power over Mathematics. Ernest (2002) claims that this will help him to become socially and epistemologically empowered and to be successful in his studies at university and to eventually. According to this model APM does ease the transition from school to university graduate successfully.

The golden thread that flows through all these analyses is that of confidence. Stankov, Morany and Lee (2013) claim that confidence on its own explains more variance in Mathematical performance than all the other self-belief constructs together. Stankov (2014) claims confidence to be the best non-cognitive predictor of mathematical achievement.

The analyses of their answers on specific questions in the questionnaire show that APM-taking taught students time-management skills, skills on how to handle large workload, better comprehension of Mathematics and gave them confidence in their Mathematical abilities. The average student who took APM had a much better experience of the transition from school Mathematics to university Mathematics than his peers who did not take APM.

Although the student who has the ability, time and determination to take an additional subject such as APM is usually the academically stronger student, the statistical data analyses show that these academically stronger students who take APM still outperform their peers in the first semester of first-year Mathematics. In the second semester these differences even out. As seen in the comparison of the content in the curricula, the APM student is introduced to new mathematical content for the first time in the second semester. The students with APM experience less anxiety in the transition from school to university, have more time to adapt to university life and have the confidence and good prior knowledge of Mathematics to handle the rigour of first-year Mathematics.

The main research question: '*To what extent does the subject Advanced Programme Mathematics prepare learners for the rigour of first-year Mathematics in the STEM university programmes?*' has been answered in this chapter.

Chapter 6 will address the conclusion, limitations, strengths and recommendations of this study.

CHAPTER 6

CONCLUSIONS

6.1 INTRODUCTION AND SUMMARY

The aim of this study was to evaluate the effectiveness of Advanced Programme Mathematics (APM) in easing the transition from high school Mathematics to university Mathematics. In South Africa many mathematically talented students currently enter higher education (HE) unprepared for university Mathematics. Although these learners do qualify for access to STEM programmes, they often lack the problem-solving, logical thinking, analysis and critical thinking skills necessary for the rigour of first-year Mathematics. Many of them do not have independent study skills and the ability to handle large workloads.

The Independent Examination Board (IEB) claims in its curriculum statement that APM provides an opportunity to improve these skills in talented and interested learners. This study investigated the validity of this claim (IEB, 2006). The research included a comparative analysis of the curricula of the National Senior Certificate (NSC), Grade 12 Mathematics, the APM course, as well as the first-year university courses at Stellenbosch University. An empirical study was also done to determine if the students who took APM were necessarily more successful in first-year Mathematics courses. A questionnaire was sent to all the first-year students in Mathematics to determine the students' perceptions of their experiences of APM and the role that APM played in their transition from school to university Mathematics.

6.2 FINDINGS AND CONCLUSIONS

The findings of this study can be summarised as follows:

- 6.2.1 The content of the APM syllabus shows similarities with large parts of the NSC syllabus, and overlaps substantially with specifically the first semester of the first-year university Mathematics curriculum. It lays a strong foundation for Calculus,

which is the main focus of the first year Mathematics syllabus. This provides the individual a measure of domain knowledge, referred to by the theory of Ackerman et al., (2013) as an important foundation of prior knowledge and skills required at entry to university and for subsequent tertiary academic success.

- 6.2.2 Because the APM syllabus covers many topics, and offers little instruction time, students are 'forced' to take responsibility for their own learning. They then in turn start evaluating their own progress in solving problems and develop the ability to make sense of their own mathematical tasks, their solutions and answers to questions. They start asking the questions themselves. This means that they become mathematically and epistemologically empowered as defined by Ernest (2002) and discussed in section 3.5.4.
- 6.2.3 The descriptions of the cognitive demands of the curricula of the NSC Mathematics curriculum and the APM curriculum seem to be very similar. A much faster work tempo is however required in the APM examination than in the NSC examination. This tempo correlates well with that expected of students in the first-year Mathematics examinations.
- 6.2.4 The APM examination paper requires a problem solving approach and a critical thinking style, as the solutions to questions require both understanding and insight. APM introduces learners to the concept of formal proof, working from definitions as well as using metacognitive skills. All of these thinking skills are necessary for further studies in Mathematics and are in accordance with the characterisation of a fully empowered student in the mathematical empowerment theory of Ernest (2002).
- 6.2.5 Learners who take APM perform significantly better than their peers in the first-semester university Mathematics, especially in the mark categories of 80-100%. This effect evened out in the second semester. This good performance is often the first step to opportunities to advanced studies and better's the student's life chances in study and work. AMP thus leads to social empowerment through Mathematics. (See the social empowerment theory of Ernest (2002) in section 3.5.3(b)).

- 6.2.6 The APM marks of students prove to be a good predictor of success in first semester Mathematics. When the APM mark, the NBT mark and the NSC Mathematics marks are combined in a general regression model, the APM mark is the best predictor of first-semester Mathematics performance. This is a new contribution to the domain of knowledge on predictors of success in first-semester university Mathematics. It confirms the theory of Ackerman, Kanfer and Beier (2013), which specifically showed the potential value of using the AP examination scores of the US in the prediction of future performance and STEM mayor persistence. (See section 3.5.4).
- 6.2.7 When the first semester mark is added to the general regression model, it outperforms the APM, NSC and NBT marks as the best predictor of success in second-semester university Mathematics.
- 6.2.8 The statistical data analysis, as well as the voice of the students indicate that APM course-taking leads to increased confidence in Mathematics and reduces mathematical anxiety, which is a good predictor of success in the first-year Mathematics. This is underpinned by Bandura's theory of self-beliefs, which culminated in the work of Stanley and Lee (2014) on confidence as the best non-cognitive predictor of success (See section 2.5.2(b)).
- 6.2.9 APM teaches students independent study skills, time management skills and how to handle large workloads. It helps them to understand first-year Mathematics and they are then able to spend more time on their other subjects. APM course-taking leads to enjoyment of Mathematics and is perceived to be challenging and intellectually stimulating.
- 6.2.10 This study confirms findings from the literature on Advanced Placement Programmes in the US and other countries of the following facts that are noteworthy for the South African APM context: a) the quality of the APM course and potential effectiveness may be compromised by a too rapid expansion of the program, b) adding an APM program to a school imposes a severe burden on a new APM teacher and increases the workload and stress levels of the academically talented learner, c) appropriate teacher assignments are vital to

the effectiveness of the programme, d) the effectiveness of the AP programme deteriorates when teachers become over- involved and learners do not work independently any more.

These findings lead to the conclusion that APM definitely eases the transition from school Mathematics to university Mathematics and thereby answers the main research question: *To what extent does the subject Advanced Programme Mathematics prepare learners for the rigour of first-year Mathematics in the STEM university programmes?*

6.3 LIMITATIONS OF THIS STUDY

The limitations in this study are:

- 6.3.1 The Engineering students and the BSc students had to be treated as one cohort (no differentiation between the groups) because of the limited sample size in the BSc Mathematics course (Mathematics 114 and Mathematics 144).
- 6.3.2 The unavailability of APM marks in official records of the SU impaired the data collection.
- 6.3.3 There was a certain percentage of non-responsiveness concerning the questionnaire because of the voluntary participation of the students; It can therefore not be verified with certainty that the resulting sample represents a random and representative sample of the target population; The results of this study may be biased and caution should be applied when generalisation to the target population is done.
- 6.3.4 The narrative data could have been explored to a deeper level if it had been possible to conduct focus group interviews after the data from the questionnaire had been analysed.
- 6.3.5 This study reported a very broad topic, and attempted three different types of data analyses. Many different notions were discussed, such as giftedness, access and success at university issues, as well as all the psychological constructs predicting success. It was therefore not possible to reach any

conclusions on gender patterns, ethnic representation or the role of the teacher in APM course-taking. Data on these issues could have presented interesting findings.

6.4 RECOMMENDATIONS

For further research:

- 6.4.1 This study was done at a high profile 'previously white' university and this study can be replicated on a larger scale by other higher educational institutions in their own unique environments.
- 6.4.2 A deeper qualitative study of the effectiveness of APM could lead to valuable insights.
- 6.4.3 The practical implications of adding an additional course to the current Mathematics curriculum still need to be examined.

To students:

- 6.4.4 If students are pursuing a career in the STEM programmes, they are strongly advised to take the subject APM. It will definitely lay a good mathematical foundation for the rigour of first year Mathematics, and will provide them with many skills necessary for success in university Mathematics.
- 6.4.5 If students want to apply and enter for any other studies at a university, it is also recommended that they take the subject APM. The cognitive skills acquired during their interaction with advanced Mathematics will provide them with the necessary edge to do well in the National Benchmark Tests, which in turn will ease access to particular selected courses. Independent study skills and dedication to hard work will make the transition to HE much easier.

To schools:

- 6.4.6 It is non-negotiable that each school with an academic focus should, where possible, give its mathematically gifted learners the opportunity to increase their mathematical skills and therefore the likelihood of graduating from university.
- 6.4.7 Access to an advanced course in Mathematics is within the control of each individual school. Although principals and teachers cannot change contextual variables, they must provide opportunities for the development of the students they have. They can encourage governing bodies of their schools to offer the opportunity of an APM course to their learners.
- 6.4.8 Schools can be innovative and creative in using local resources. If a school has limited resources, schools in the same neighbourhood can share a teacher and present classes at a central point.
- 6.4.9 Individuals in small schools with minimum teaching resources or schools in rural areas can do the APM course through distance learning or online through virtual means.

To curriculum planners:

- 6.4.10 It will definitely make sense to reintroduce a course similar to the APM course in the school syllabus in an attempt to alleviate the critical shortage of STEM graduates in South Africa.

To teachers:

- 6.4.11 It is important that teachers are conscious of their power in influencing students. Mathematics teachers and guidance councillors must take up the responsibility of encouraging students who want to pursue a career in STEM to enrol for APM.
- 6.4.12 Students will only benefit from APM course-taking as long as it remains a course where students have to take responsibility for their own learning and are left to work independently. When teachers start 'coaching' and taking control of the

learning, APM will lose its unique character and the specific value that it adds to gifted students' learning experience in Mathematics.

6.5 A FINAL WORD

The researcher's work as an educator makes her determined to promote APM as a course that instils confidence and prepares students for an easy transition from school to first-year Mathematics.

This study concludes with two quotations:

The first one is defined as the purpose of APM in the curriculum document (IEB, 2006):

The study of Advanced Programme Mathematics contributes to the personal development of high performing mathematics learners by providing challenging learning experiences; feelings of success and self-worth, and to the development of appropriate values and attitudes through the successful application of its knowledge and skills in context, and through the collective engagement with mathematical ideas (p. 11).

The researcher is convinced that these claims have been verified.

The second one is voice of an APM student in the researcher's Grade 12 class of 2013:

The most important aspect of APM for me is that it taught me another way of working and thinking. It's not the Mathematics itself that helped me at university, but the way of thinking and learning that APM taught.

.....

REFERENCES

- Ackerman, P. L. (2005). Ability determinants of individual differences in skilled performance. In R.J. Sternberg & J. E. Pretz (Eds.), *Cognition and Intelligence: Identifying the Mechanisms of the Mind*, (pp. 42-159). New York, NY: Cambridge University Press.
- Ackerman, P. L., Bowen, K. R., Beier, M., & Kanfer, R. (2001). Determinants of individual differences and gender differences in knowledge. *Journal of Educational Psychology*, 93(4), 797- 825.
- Ackerman, P.L., & Heggestad, E.D. (1997). Intelligence, personality, and interests: Evidence for overlapping traits. *The Psychological Bulletin*, 121(2), 219 - 245.
- Ackerman, P. L., Kanfer, R., & Beier, M. E. (2013). Trait complex, cognitive ability, and domain knowledge predictors of baccalaureate success, STEM persistence, and gender differences. *Journal of Educational Psychology*, 105(3), 911- 927.
- Ackerman, P. L., Chamorro-Premuzic, T., & Furnham, A. (2011). Trait complexes and academic achievement: Old and new ways of examining personality in educational contexts. *British Journal of Educational Psychology*, 81(1), 27-40.
doi:10.1348/000709910X522564
- Ackerman, P. L., Kanfer, R., & Calderwood, C. (2013). High school advanced placement and student performance in college: STEM majors, non-STEM majors, and gender differences. *Teachers College Record*, 115(10), 1-43.

- Adelman, C. (2006). *The Toolbox Revisited: Paths to Degree Completion from High School Through College*. Washington, D.C.: US Department of Education.
- Adler, J., & Setati, M. (2005). Mathematics as filter of equity - an 'old' story and a new telling. Response to Michael Kahn: A class act - Mathematics as filter of equity in South Africa's schools. *Perspectives in Education*, 23(3), 149-152.
- Allport, G. W. (1927). Concepts of trait and personality. *Psychological Bulletin*, 24(5), 284-293. doi:10.1037/h0073629
- Alreck, P. L., & Settle, R. B. (2004). *The survey research handbook*. Boston: McGraw-Hill/Irwin.
- Ampadu, E., & Adofo, S. (2014). The importance of theoretical underpinning for a school mathematics curriculum: The Ghanaian experience. *International Journal of Research Studies in Education*, 3(3), 107-118.
- Arcavi, A. (2000). Problem-driven research in Mathematics education. *The Journal of Mathematical Behavior*, 19(2), 141-173.
- Astin, A. W. (1984). Student Involvement: A Developmental Theory for Higher Education. *Journal of College Student Personnel*, 25(4), 297-308.
- Astin, A. W. (1993). *What matters in college?: Four critical years revisited*. (Vol.1). San Francisco: Jossey-Bass.

- Attridge, N., & Inglis, M. (2013). Advanced mathematical study and the development of conditional reasoning skills. *PloS ONE*, 8(7), e69399.
doi:10.1371/journal.pone.0069399
- Babbie, E., & Mouton, J. (2001). The ethics and politics of social research. *The Practice of Social Research*. Oxford: Oxford University Press.
- Bailyn, B. (1972). *Education in the forming of American society: Needs and opportunities for study*. Chapel Hill: University of North Carolina Press.
- Baker, S. R. (2004). Intrinsic, extrinsic, and amotivational orientations: Their role in University adjustment, stress, well-being, and subsequent academic performance. *Current Psychology*, 23(3), 189-202.
- Bandura, A. (1977). Self-efficacy: Toward a unifying theory of behavioral change. *Psychological Review*, 84(2), 191.
- Bandura, A. (1986). *Social foundations of thought and action: A Social cognitive Theory*. Englewood Cliffs, NJ: Prentice-Hall.
- Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.
- Bandura, A. (1999). Social cognitive theory of personality. In L. Pervin & O. John (Ed.), *Handbook of personality* (2nd ed., pp. 154-196). New York: Guilford Publications. (Reprinted in D. Cervone & Y. Shoda [Eds.], *The coherence of personality*. New York, NY: Guilford Press.)

- Bandura, A., & Schunk, D. H. (1981). Cultivating competence, self-efficacy, and intrinsic interest through proximal self-motivation. *Journal of Personality and Social Psychology*, 41(3), 586.
- Bean, J. P. (1980). Dropouts and turnover: The synthesis and test of a causal model of student attrition. *Research in Higher Education*, 12(2), 155-187.
- Berger, M., Bowie, L., & Nyaumwe, L. (2010). Taxonomy matters: Cognitive levels and types of mathematical activities in Mathematics examinations. *Pythagoras*, (71), 30-40.
- Bergqvist, E. (2007). Types of reasoning required in university exams in Mathematics. *The Journal of Mathematical Behaviour*, 26(4), 348-370.
- Berry, J. M. (1987). *A self-efficacy model of memory performance*. Paper presented in Annual Meeting of the American Educational Research Association, New York.
- Bhorat, H., Tseng, D., & Stanwix, B. (2014). Pro-poor growth and social protection in South Africa: Exploring the interactions. *Development Southern Africa*, 31(2), 219-240.
- Biggs, J. (1996). Enhancing teaching through constructive alignment. *Higher Education*, 32(3), 347-364.
- Biggs, J. (1999). What the student does: Teaching for enhanced learning. *Higher Education Research & Development*, 18(1), 57-75.

- Binet, A., & Simon, T. (1916). *The development of intelligence in children: The Binet-Simon scale*. Baltimore: Williams & Wilkens.
- Bleske-Rechek, A., Lubinski, D., & Benbow, C. P. (2004). Meeting the educational needs of special populations. *Psychological Science*, 15(4), 217-224.
- Bloom, B. S. (1956). *Taxonomy of educational objectives. (Vol. 1). Cognitive domain*. New York, NY: McKay.
- Boazman, J., & Sayler, M. (2011). Personal well-being of gifted students following participation in an early college-entrance program. *Roeper Review*, 33(2), 76-85. doi:10.1080/02783193.2011.554153
- Boeije, H. R. (2009). *Analysis in Qualitative Research*. Thousand Oaks, CA: Sage Publications.
- Bogdan, R. C., & Biklen, S. K. (1998). *Qualitative research in education. An introduction to theory and methods*. Boston: Allyn and Bacon.
- Bong, M., Cho, C., Ahn, H. S., & Kim, H. J. (2012). Comparison of self-beliefs for predicting student motivation and achievement. *The Journal of Educational Research*, 105(5), 336-352.
- Bottoms, G., & Feagin, C. (1997). *The 1996 high schools that work assessment: Science good news, bad news and actions*. (Research Report of Southern Regional Education Board), 4, 1–14.

- Bouffard-Bouchard, T. (1990). Influence of self-efficacy on performance in a cognitive task. *The Journal of Social Psychology*, 130(3), 353-363.
- Bouffard-Bouchard, T., Parent, S., & Lavirée, S. (1993). Self-regulation on a concept-formation task among average and gifted students. *Journal of Experimental Child Psychology*, 56(1), 115-134.
- Bowen, W. G., Chingos, M. M., & McPherson, M. S. (2009). *Crossing the finish line: Completing College at America's public universities*. Princeton, N J: University Press.
- Bowen, W. G., Kurzweil, M. A., Tobin, E. M., & Pichler, S. C. (2006). *Equity and Excellence in American Higher Education*. Charlottesville, VA: University of Virginia Press.
- Boyatzis, R. E. (1998). *Transforming qualitative information: Thematic analysis and code development*. Thousand Oaks, CA: Sage Publishers.
- Bragdon, H. W. (1960). Advanced Placement: Rising tide with breakers ahead. *College Board Review*, 42, 18-20.
- Bray, R., Gooskens, I., Kahn, L., Moses, S., & Seekings, J. (2010). *Growing up in the new South Africa: Childhood and adolescence in post-apartheid Cape Town*. Cape Town: Human Sciences Research Council Press.
- Bressoud, D. M. (2004). The changing face of Calculus: First-and second-semester calculus as a high school course. In C. L. Diefenderfer & R.B. Nelson (Eds.), *Calculus Collection: A Resource for AP and Beyond* (pp.54-58). Washington DC: MAA Service Centre.

- Buice, S. M. (2012). *Why students are choosing not to participate in honors and advanced placement classes: The phenomenon in one Georgia high school*. Unpublished doctoral Dissertation, Liberty University, Georgia.
- Cambridge International Examinations. (2014). *Cambridge International AS and A Levels*. Retrieved from <http://www.cie.org.uk/programmes-and-qualifications/cambridge-advanced/cambridge-international-as-and-a-levels/>
- Carpenter, P. A., Just, M. A., & Shell, P. (1990). What one intelligence test measures: A theoretical account of the processing in the Raven Progressive Matrices Test. *Psychological Review*, 97(3), 404.
- Casement, W. (2003). Declining credibility for the AP program. *Academic Questions*, 16(4), 11-25.
- Casserly, P. L. (1986). Advanced placement revisited. *ETS Research Report Series*, 1986(2), 1-24.
- Chajewski, M., Mattern, K. D., & Shaw, E. J. (2011). Examining the role of advanced placement exam participation in 4-year college enrolment. *Educational Measurement: Issues & Practice*, 30(4), 16-27. doi:10.1111/j.1745-3992.2011.00219.x

- Chen, C., Lee, S., & Stevenson, H. W. (1996). Academic achievement and motivation of Chinese students: A cross-national perspective. In S. Lau (Ed.), *Growing Up the Chinese Way: Chinese Child and Adolescent Development* (pp. 69 -91). Hong Kong: Chinese University of Hong Kong Press.
- Chisholm, L. (2003). The state of curriculum reform in South Africa: The issue of Curriculum 2005. In J Daniel, A Habib & R Southall (Eds.), *State of the Nation. South Africa 2003-2004* (pp. 268-289). Cape Town: Human Sciences Research Council Press.
- Cho, S., & Lin, C. (2010). Influence of family processes, motivation, and beliefs about intelligence on creative problem solving of scientifically talented individuals. *Roeper Review*, 33(1), 46-58. doi:10.1080/02783193.2011.530206
- Clark, M., & Lovric, M. (2009). Understanding secondary–tertiary transition in Mathematics. *International Journal of Mathematical Education in Science and Technology*, 40(6), 755-776.
- Clemons, S. A. (2006). Constructivism pedagogy drives redevelopment of CAD course: A case study: Rather than feeding information to the student through direct instruction, the teacher is maintaining the role of facilitator in the learning process. *The Technology Teacher*, 65(5), 19.
- Cobb, P. (2007). Putting philosophy to work. *Second Handbook of Research on Mathematics Teaching and Learning: A Project of the National Council of Teachers of Mathematics*, 1, 1.

Cohen, L. M., & Manion, L. (2000). *Research Methods in Education*, 5, 181-190.

Colangelo, N., Assouline, S. G., & Gross, M. U. (2004). *A Nation Deceived: How Schools Hold Back America's Brightest Students*. (Templeton National Report on Acceleration). (Vol.2). Iowa, IA: Connie Belin & Jacqueline N. Blank International Center for Gifted Education and Talent Development.

College Board. (2014). *AP Calculus AB Course Home Page*.

<http://www.collegeboard.com/>

Craig, T. S. (2013). Conceptions of mathematics and student identity: Implications for engineering education. *International Journal of Mathematical Education in Science and Technology*, 44(7), 1020-1029. doi:10.1080/0020739X.2013.823521

Creswell, J. W. (2013). *Research design: Qualitative, quantitative, and mixed methods approaches*. Thousand Oaks, CA: Sage Publications.

Creswell, J. W., & Garrett, A. L. (2008). The "movement" of mixed methods research and the role of educators. *South African Journal of Education*, 28(3), 321-333.

Creswell, J. W., & Plano Clark, V. (2007). Choosing a mixed methods design. In J.W Creswell & V Plano Clark, *Designing and Conducting Mixed Methods Research* (pp. 58-88). Thousand Oaks, CA: Sage Publications.

Croucamp, A. (2008). *Thinking skills*. Paper presented at Mindburst Workshop at Sacred Heart College, Johannesburg.

- Cross, T. L., Coleman, L. J., & Stewart, R. A. (1993). The social cognition of gifted adolescents: An exploration of the stigma of giftedness paradigm. *Roeper Review*, 16(1), 37-40.
- Cross, M., Mungadi, R., & Rouhani, S. (2002). From policy to practice: Curriculum reform in South African education. *Comparative Education*, 38(2), 171-187.
doi:10.1080/03050060220140566
- Crouch, L., & Vinjevold, P. (2006). South Africa: Access before quality and what to do now? *Profesorado-Revista De Curriculum y Formacion Del Profesorado*, 10(1), 1-16.
- Curry, W., MacDonald, W., & Morgan, R. (1999). The Advanced Placement Program: Access to excellence. *Journal of Secondary Gifted Education*, 11(1), 17-23.
- Datta, L. (1994). Paradigm wars: A basis for peaceful coexistence and beyond. *New Directions for Program Evaluation*, (61), 53-70.
- De Beer, M., & Van der Merwe, D. (2006). Challenges of student selection: Predicting academic performance. *South African Journal of Higher Education*, 20(4), 547-562.
- DBE (Department of Basic Education). (2010). *Guidelines for Exclusive Teaching and Learning*. Pretoria: Government Printer.
- DBE (Department of Basic Education). (2011). *Curriculum and Assessment Policy Statement (CAPS) Grades 10-12, Mathematics*. Pretoria: Department of Basic Education.

DBE (Department of Basic Education). (2012). *National Protocol for Assessment Grades R -12*. Pretoria: Department of Basic Education.

DBE (Department of Basic Education). (2013). *Annual report 2012/2013*. Pretoria: Department of Basic Education.

Demetriou, C., & Schmitz-Sciborski, A. (2011). Integration, motivation, strengths and optimism: Retention theories past, present and future. In R. Hayes (Ed.), *Proceedings of the 7th National Symposium on Student Retention, 2011, Charleston* (pp. 300-312). Norman, OK: The University of Oklahoma.

Denzin, N. K. (2010). Moments, mixed methods, and paradigm dialogs. *Qualitative Inquiry*, 16(6), 419-427.

DoE (Department of Education). (1995). White Paper on Education and Training. *Government Gazette* 357 (pp. 4-80). Pretoria: Government Printer.

DoE (Department of Education). (2001). *Education White Paper 6. Special Needs Education. Building an Inclusive Education and Training System*. Pretoria: Government Printer.

DoE (Department of Education). (2001). *National Plan for Higher Education*. Pretoria: Government Printer.

DoE (Department of Education). (2003). *National Curriculum Statement Grades 10-12 (General) Policy: Mathematics content*. Pretoria: Government Printer.

- DoE (Department of Education). (2006). *National Curriculum Statement Grades 10-12 (General): Advanced Programme Mathematics*. Pretoria: Department of Education.
- Dougherty, C., Mellor, L., & Jian, S. (2006). *The relationship between advanced placement and college graduation*. (Report 1). Retrieved from <http://files.eric.ed.gov/fulltext/ED519365.pdf>
- Du Plessis, L., & Gerber, D. (2012). Academic preparedness of students - an exploratory study. *TD : The Journal for Trans- disciplinary Research in Southern Africa*, 8(1), 81-94.
- Du Preez, J., Steyn, T., & Owen, R. (2008). Mathematical preparedness for tertiary Mathematics- a need for focused intervention in the first year? *Perspectives in Education*, 26(1), 49-62.
- Eccles, J., & Wigfield, A. (1985). Teacher expectations and student motivation. In D.B. Dusek. *Teacher Expectancies* (pp. 185-226). Hillsdale, NJ: Erlbaum.
- Eccles, J. S., & Wigfield, A. (2002). Motivational beliefs, values, and goals. *Annual Review of Psychology*, 53(1), 109.
- Edwards, N. (2010). An analysis of the alignment of the grade 12 physical sciences examination and the core curriculum in South Africa. *South African Journal of Education*, 30(4), 571-590.
- Eiselen, R. J. (2006). *Predicting achievement in Mathematics at tertiary level* Unpublished doctoral dissertation. Johannesburg: University of Johannesburg.

- Engelbrecht, J., Harding, A., & Phiri, P. (2010). Are OBE-trained learners ready for university mathematics? *Pythagoras*, (72), 3 -13.
- English, F. W. (2000). *Deciding what to teach and test: Developing, aligning, and auditing the curriculum*. Thousand Oaks, CA: Corwin Press.
- Ernest, P. (2002). Empowerment in Mathematics Education. *Philosophy of Mathematics Journal*, 15, 1–16.
- Ernest, P. (2010). Commentary 2 on reflections on theories of learning. In B.Sriraman & L. English, *Theories of Mathematics Education* (pp. 53-61). Heidelberg: Springer.
- Eyre, D., & Lowe, H. (2013). *Curriculum provision for the Gifted and Talented in the Secondary School*. New York, NY: Routledge.
- Farrell, M. A., & Farmer, W. A. (1980). *Systematic instruction in Science for the middle and high school years*. Reading, MA: Addison-Wesley Publishing Company.
- Feilzer, M. Y. (2010). Doing mixed methods research pragmatically: Implications for the rediscovery of pragmatism as a research paradigm. *Journal of Mixed Methods Research*, 4(1), 6-16.
- Felder, R. M., & Brent, R. (2001). Effective strategies for cooperative learning. *Journal of Cooperation & Collaboration in College Teaching*, 10(2), 69-75.
- Fisher, G., & Scott, I. (2011). The role of Higher Education in closing the skills gap in South Africa. *Background Paper 3: Closing the skills and technology gap in South Africa*. Cape Town: The World Bank.

- Fishman, M. (1978). Crime waves as ideology. In P. A. Quiroz & N. Flores-Gonzalez. *Social Problems* (pp. 531-543). Doi: <http://dx.doi.org/10.2307/800102>
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. *The Nature of Intelligence*, 12, 231-235.
- Fleisch, B. (2008). *Primary education in crisis: Why SA school children underachieve in Reading and Maths*. Cape Town: Juta & Co. Ltd.
- Flowers, L. A. (2008). Racial differences in the impact of participating in advanced placement programs on educational and labour market outcomes. *Educational Foundations*, 22, 121-132.
- Ford, D. Y., Grantham, T. C., & Whiting, G. W. (2008). Another look at the achievement gap learning from the experiences of gifted black students. *Urban Education*, 43(2), 216-239.
- Foust, R. C., Hertberg-Davis, H., & Callahan, C. M. (2009). Students' perceptions of the non-academic advantages and disadvantages of participation in advanced placement courses and international baccalaureate programs. *Adolescence*, 44(174), 289-312.
- Frick, B. L. (2008). *The profile of the Stellenbosch university first-year student: Present and future trends*. (Unpublished Research Report, Centre for Teaching and Learning, Stellenbosch University, South Africa). Retrieved from <Http://stbweb02.Stb.Sun.Ac.za/ctl/documents/Student>

- Furnham, A., & Chamorro-Premuzic, T. (2004). Personality and intelligence as predictors of statistics examination grades. *Personality and Individual Differences*, 37(5), 943-955.
- Furnham, A., Chamorro-Premuzic, T., & McDougall, F. (2002). Personality, cognitive ability, and beliefs about intelligence as predictors of academic performance. *Learning and Individual Differences*, 14(1), 47-64.
- Gallagher, J. J. (2005). According to Jim Gallagher: The role of race in gifted education. *Roeper Review*, 27(3), 135-135.
- Gardner, H. (1993). *Multiple intelligences: The theory in practice*. New York, NY: Basic Books.
- Geake, J. G., & Gross, M. U. (2008). Teachers' negative affect toward academically gifted students an evolutionary psychological study. *Gifted Child Quarterly*, 52(3), 217-231.
- Geiser, S., & Santelices, V. (2004). The role of advanced placement and honors courses in college admissions. In P. Gandara, G. Orfield, C. Horn, *Expanding Opportunity in Higher Education: Leveraging Promise* (pp. 75-113). New York, NY: SUNY Press.
- Gibbon, T. (2010). Weighing success and diversity in the balance at Stellenbosch university. In M. Letseka, M. Cosser, M. Breier, M. Visser. *Student Retention and Graduate Destination. Higher Education and Labour Market Access and Success*, (pp. 67-86). Cape Town: Human Sciences Research Council Press.

- Given, L. M. (2008). *The Sage Encyclopaedia of Qualitative Research Methods*. Thousand Oaks, CA: Sage Publications.
- Goodlad, J. I. (1979). *Curriculum inquiry. The study of curriculum practice*. New York: McGraw-Hill.
- Gorard, S. (2013). *Research design: Creating robust approaches for the social sciences*. Thousand Oaks, CA: Sage Publications.
- Govender, J., & Moodley, M. (2012). Effect of National Senior Certificate Maths on the pass rate of first-year Engineering Physics: Research article. *South African Journal of Science*, 108(7), 1-5.
- Green, G., & Rollnick, M. (2007). Let's stop complaining about low cognitive levels of testing in time-limited examinations: The case for fair testing practices. *South African Journal of Higher Education*, 21(2), 255-266.
- Greene, J. C. (2007). *Mixed methods in social inquiry*. San Francisco, CA: Jossey-Bass.
- Griesel, H. (2006). Access and entry level benchmarks: The national benchmark tests project. *South African Journal of Higher Education*, 21(5), 610-616.
- Gross, M. U. (2002). Social and emotional issues for exceptionally intellectually gifted students. In M. Neihart, S. Reis, N. Robinson & S. Moon. *The Social and Emotional development of Gifted Children: What do we know* (pp. 193–204). Waco, TX: Prufrock Press.

- Gueudet, G. (2008). Investigating the secondary–tertiary transition. *Educational Studies in Mathematics*, 67(3), 237-254.
- Hackett, G., & Betz, N. E. (1995). Self-efficacy and career choice and development. In J. E. Maddux, *Self-efficacy, adaptation, and adjustment* (pp. 249-280). New York, NY: Springer Science+ Business Media.
- Hampel, R. L. (1986). *The last little citadel: American high schools since 1940*. New York, NY: John Wiley Publishers.
- Hertberg-Davis, H. (2008). A narrow escape: Gifted students' perceptions of advanced placement and international baccalaureate programs. *The Gifted Child Newsletter*, 52(3), 199-216.
- Hertberg-Davis, H., Callahan, C. M., & Kyburg, R. M. (2006). *Advanced placement and international baccalaureate programs: A "fit" for gifted learners?* (Research Report: National Research Centre on the Gifted and Talented). Retrieved from <http://www.gifted.uconn.edu/nrcgt.html>
- Hill, T., & Lewicki, P. (2012). *Electronic Statistics Textbook*. Tulsa, OK: Stat Soft Inc. Retrieved from: <http://www.Statsoft.com/textbook>
- Hlalele, D. (2010). Do learning skills acquired in the university access programme enhance participation in academic practice? *South African Journal of Higher Education*, 24(1), 98-110.
- Hobden. (2002). Reclaiming the meaning of problem solving: The need for a common understanding of the terms problem and problem solving. *Proceedings of the Tenth*

Annual Meeting of the Southern African Association of Research in Mathematics, Science and Technology Education (pp. 98-204). Durban: University of Natal.

Hochman, W. R. (1970). Advanced placement: Can it change with the times? *College Board Review*, 77(3), 16-19.

Holborn, L. (2013). Time to take the shine off the school story, fast facts. Retrieved from <http://irr.org.za/reports-and-publications/fast-facts/fast-facts-2013>

Howson, A., Barnard, A., Crighton, D., Davies, N., Gardiner, A., Jagger J., Steele, N. (1995). *Tackling the mathematics problem*. London: London Mathematical Society, the Institute of Mathematics and its Applications and the Royal Statistical Society.

IEB (Independent Examination Board). (2012). *Advanced programme mathematics Funding 2012 circular 119/2011*. Retrieved from <http://ieb.co.za/>

IEB (Independent Examination Board). (2011). *IEB Assessment Matters*. Retrieved from <http://ieb.co.za/School/FAQs/Universities/index.html>

IEB (Independent Examination Board). (2007). *The IEB Advanced Programme Courses*, Retrieved from [http://doc.ieb.co.za/view.php?f document ID = 24425](http://doc.ieb.co.za/view.php?f=document+ID+24425)

IEB (Independent Examination Board). (2006). *APM Curriculum Statements Grades 10-12 (General)*. Johannesburg: Independent Examination Board.

IEB. (n.d.). *Advanced Programme Mathematics (Press Release)* Johannesburg: Independent Examination Board.

International Baccalaureate Diploma (2014). *Diploma-Programme*. Retrieved from:
<http://www.ibo.org/en/programmes/diploma-programme/what-is-the-dp/>

Jackson, S. (2011). *Research methods and statistics: A critical thinking approach*.
Boston, MA: Cengage Learning.

Jacobs, M. (2010). *A Framework for the placement of University Students in science Programmes*. Unpublished doctoral dissertation. Bloemfontein: University of the Free State.

Jaeger, A., Bresciani, M., & Sabourin, C. (2002). Predicting persistence and academic performance of first year students: An assessment of emotional intelligence and non-cognitive variables. *Communications of North Carolina State University*. Retrieved from www.researchgate.net

Jankvist, U. T. (2011). [Review of the book *Theories of mathematics education: Seeking new frontiers (Advances in mathematics education)* by B. Sriraman & L. English (eds.) (2010)]. *Mathematical thinking and learning*, 13(3), 247-257.

Jansen, J. D. (1998). Curriculum reform in South Africa: A critical analysis of outcomes-based education [1]. *Cambridge Journal of Education*, 28(3), 321-331.

Jansen, J. D. (1999). The school curriculum since apartheid: Intersections of politics and policy in the South- African transition. *Journal of Curriculum Studies*, 31(1), 57-67.

Jansen J.D. (2013). Personal reflections on policy and school quality in South Africa: When the politics of disgust meet the politics of distrust. In Y. Sayed, A. Kanjee, M .

- Nkomo, *The search for quality education in post-apartheid South Africa: Interventions to improve learning and teaching* (pp. 81-89). Cape Town: Human Sciences Research Council Press.
- Jansen, J. D., & Christie, P. (Eds.). (1999). *Changing curriculum: Studies on outcomes-based education in South Africa*. Cape Town: Juta & Co Ltd.
- Jansen, J. D., & Sayed, Y. (Eds.). (2001). *Implementing education policies: The South African experience*. Cape Town: UCT Press.
- Jonathan, R. (2000, October). *The role of schooling in social transformation: High hopes and reasonable expectations*. Paper presented at the Kenton Ebhayi Conference. Port Elizabeth, South Africa.
- Joyce, T. B. Y., & Yates, S. M. (2007). A Rasch analysis of the academic self-concept questionnaire. *International Education Journal*, 8(2), 470-484.
- Kell, H. J., Lubinski, D., & Benbow, C. P. (2013). Who rises to the top? Early indicators. *Psychological Science*, 24(5), 648-659. doi:10.1177/0956797612457784
- Keng, L., & Dodd, B. G. (2008). *A comparison of college performances of AP and non-AP student groups in 10 subject areas*. Princeton, NJ: The College Board.
- Kilpatrick, J. (2010). Preface to part 1. In B. Sriraman, & I. English (Eds.), *Theories of Mathematics Education: Seeking new frontiers (Advances in Mathematics Education* (pp. 3-5). Heidelberg: Springer.

- Klopfenstein, K. (2004). The advanced placement expansion of the 1990s: How did traditionally underserved students fare? *Education Policy Analysis Archives*, 12(68). Retrieved from: <http://epaa.asu.edu/ojs/index.php/epaa/article/view/223>
- Klopfenstein, K., & Thomas, M. K. (2005). *The advanced placement performance advantage: Fact or fiction*. Retrieved from https://www.aeaweb.org/assa/2005/0108_1015_0302.pdf
- Klopfenstein, K., & Thomas, M. K. (2010). Advanced placement participation: Evaluating the policies of states and colleges. In P.M. Sadler, G. Sonnert, R.H. Tai, K. Klopfenstein, *AP: A Critical Examination of the Advanced Placement Program* (pp. 167-188). Cambridge: Harvard Education Press.
- Kokot, S. (1999). *Help - Our child is gifted: Guidelines for parents of gifted children*. Pretoria: Radford House.
- Krathwohl, D. R. (2002). A revision of Bloom's taxonomy: An overview. *Theory into Practice*, 41(4), 212-218.
- Kriek, J., & Basson, I. (2008). Implementation of the new FET physical science curriculum: Teachers views. *Journal of Research in Mathematics, Science and Technology Education*, 12, 63-75.
- Kuh, G. D. (2003). What we're learning about student engagement from NSSE: Benchmarks for effective educational practices. *Change: The Magazine of Higher Learning*, 35(2), 24-32.

- Kuh, G. D. (2007). How to help students achieve. *Chronicle of Higher Education*, 53(41), 12-13.
- Labuschagne, A. (2003). Qualitative research - airy fairy or fundamental. *The Qualitative Report*, 8(1), 100-103.
- Lam, D., Ardington, C., & Leibbrandt, M. (2011). Schooling as a lottery: Racial differences in school advancement in urban South Africa. *Journal of Development Economics*, 95(2), 121-136.
- Larkin, S. (2009). *Metacognition in young children*. New York, NY: Routledge.
- Lee, J. (2009). Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. *Learning and Individual Differences*, 19(3), 355-365.
- Lee, J., & Stankov, L. (2012). Special issue on non-cognitive psychological processes and academic achievement. *Educational Psychology*, 32(4), 551-552.
doi:10.1080/01443410.2012.690186
- Leech, N. L., Dellinger, A. B., Brannagan, K. B., & Tanaka, H. (2010). Evaluating mixed research studies: A mixed methods approach. *Journal of Mixed Methods Research*, 4(1), 17-31.
- Leibowitz, B., & Bozalek, V. (2014). Access to higher education in South Africa. *Widening Participation and Lifelong Learning*, 16(1), 91-109.

- Lent, R. W., Brown, S. D., & Larkin, K. C. (1986). Self-efficacy in the prediction of academic performance and perceived career options. *Journal of Counselling Psychology*, 33(3), 265.
- Lesh, R., Sriraman, B., & English, L. (2013). Theories of learning mathematics. In S. Lerman (Ed.), *Encyclopaedia of Mathematics Education* (pp. 615-623). Heidelberg: Springer.
- Lester, F. K. (2005). On the theoretical, conceptual, and philosophical foundations for research in mathematics education. *ZDM*, 37(6), 457-467.
- Liang, L. L., & Yuan, H. (2008). Examining the alignment of Chinese national physics curriculum guidelines and 12th-grade exit examinations: A case study. *International Journal of Science Education*, 30(13), 1823-1835.
- Lincoln, Y. S., & Guba, E. G. (1994). Competing paradigms in qualitative research. *Handbook of Qualitative Research*, 105-117.
- Locke, E. A., & Latham, G. P. (2006). New directions in goal-setting theory. *Current Directions in Psychological Science*, 15(5), 265-268. doi:10.1111/j.1467-8721.2006.00449.x
- Lubinski, D. (2004). Introduction to the special section on cognitive abilities: 100 years after Spearman's (1904) "general intelligence, objectively determined and measured". *Journal of Personality and Social Psychology*, 86(1), 96.
- Ma, X., & Johnson, W. (2008). Mathematics as the critical filter: Curricular effects on gendered career choices. In H. M. G. Watt, J. S. Eccles (Eds.), *Gender and*

occupational outcomes: Longitudinal assessments of individual, social, and cultural influences (pp. 55-83). Washington, D.C.: American Psychological Association.

Mackenzie, N., & Knipe, S. (2006). Research dilemmas: Paradigms, methods and methodology. *Issues in Educational Research*, 16(2), 193-205.

Marais, H. (2011). *South Africa pushed to the limit: The political economy of change*. Cape Town: UCT Press.

Mattern, K. D., Xiong, X., & Shaw, E. J. (2009). *The relationship between AP exam performance and college outcomes*. (Research Report). New York, NY: College Board.

McArthur, T. (1999). World English. *Encarta World English Dictionary*. New York, NY: St Martin's Press.

McMillan, J. H. (2001). *Classroom assessment: Principles and Practice for Effective Instruction*. (2nd ed). Needham Heights: Allyn & Bacon.

McMillan, J. H., & Schumacher, S. (2006). *Research in Education*: (6th ed.) Boston: Pearson Education.

Meece, J. L., Wigfield, A., & Eccles, J. S. (1990). Predictors of math anxiety and its influence on young adolescents' course enrolment intentions and performance in Mathematics. *Journal of Educational Psychology*, 82(1), 60.

- Mertens, D. (2005). *Research methods in education and psychology: Integrating diversity with quantitative, qualitative, and mixed methods*. Thousand Oaks, CA: Sage Publications.
- Mhlolo, M. K. (2011). *From coherence in theory to coherence in practice: A stock-take of the written, tested and taught national curriculum statement for mathematics (NCSM) at further education and training (FET) level in South Africa*. Unpublished doctoral dissertation. Johannesburg: University of the Witwatersrand.
- Migiro, S., & Magangi, B. (2011). Mixed methods: A review of literature and the future of the new research paradigm. *African Journal of Business Management*, 5(10), 3757-3764.
- Miller, E. M. (2009). The effect of training in gifted education on elementary classroom teachers' theory-based reasoning about the concept of giftedness. *Journal for the Education of the Gifted*, 33(1), 65-105.
- Mo, L., Yang, F., Hu, X., Calaway, F., & Nickey, J. (2011). ACT test performance by advanced placement students in Memphis city schools. *The Journal of Educational Research*, 104(5), 354-359.
- Mohamed, H. (1998). The implementation of OBET in South to success or recipe for failure. *Education Practise*, 1, 3-16.
- Monks, F. J., & Katzko, M. W. (2005). Giftedness and gifted education. In R. J. Sternberg, R. J. & J. E. Davidson (Eds.). *Conceptions of Giftedness*, (pp. 187-200). Cambridge: Cambridge University Press.

- Moore, D. S., & McCabe, G. P. (1989). *Introduction to the practice of Statistics*. New York, NY: WH Freeman/Times Books/Henry Holt & Co.
- Moore, G. W., & Slate, J. R. (2008). Who's taking the advanced placement courses and how are they doing: A state-wide two-year study. *High School Journal*, 92(1), 56-67.
- Morgan, R., & Crone, C. (1993). *Advanced placement examinees at the University of California: An examination of the freshman year courses and grades of examinees in biology, calculus AB, and chemistry* (ETS Statistical Report No. 93-210). Princeton, NJ: Educational Testing Service.
- Morony, S., Kleitman, S., Lee, Y. P., & Stankov, L. (2013). Predicting achievement: Confidence vs self-efficacy, anxiety, and self-concept in Confucian and European countries. *International Journal of Educational Research*, 58, 79-96.
- Morrow, W. E. (2009). *Bounds of democracy: Epistemological access in higher education*. Cape Town: Human Sciences Research Council Press.
- Muller, J. (2014). Every picture tells a story: Epistemological access and knowledge. *Education as Change*, 18(2), 255-269.
- Murphy, D., & Dodd, B. G. (2009). *A comparison of college performance of matched AP and non-AP student groups*. (Research Report). Austin, TX: College Board.
- Ndlovu, M. (2013). Revisiting the efficacy of constructivism in mathematics education. *Philosophy of Mathematics Education Journal*, 28.

- Ndura, E., Robinson, M., & Ochs, G. (2003). Minority students in high school advanced placement courses: Opportunity and equity denied. *American Secondary Education*, 32(1), 21.
- Neisser, U., Boodoo, G., Bouchard Jr, T. J., Boykin, A. W., Brody, N., Ceci, S. J., Sternberg, R. J. (1996). Intelligence: Knowns and unknowns. *American Psychologist*, 51(2), 77.
- Nel, C. (2007). *Die oorgang van skool na universiteit: 'n gevallestudie aan die universiteit Stellenbosch*. (Research Report). Stellenbosch: Stellenbosch University.
- Nel, C., & Kistner, L. (2009). The National Senior Certificate: Implications for access to higher education. *South African Journal of Higher Education*, 23(5), 953-973.
- Nel, C., Kistner, L., & E Van der Merwe. (2013). Using enrolment trends to facilitate access. *Journal of Higher Education Policy & Management*, 35(1), 85-94.
doi:10.1080/1360080X.2013.748480
- Neuman, W. L., (2006). *Social research methods: Qualitative and quantitative approaches*. Boston, MA : Allyn & Bacon.
- Niss, M. (2007). *The concept and role of theory in Mathematics education. Relating practice and research in mathematics education*. Paper presented at Norma (pp. 97-110), Roskilde, Denmark
- Nugent, S. A., & Karnes, F. A. (2002). The advanced placement program and the international baccalaureate programme: A history and update. *Gifted Child Today*, 25(1), 30-39.

- Onwuegbuzie, A. J., & Johnson, R. B. (2006). The validity issue in mixed research. *Research in the Schools*, 13(1).
- Oswald, M., & de Villiers, J. (2013). Including the gifted learner: Perceptions of South African teachers and principals. *South African Journal of Education*, 33(1), 1-21.
- Paek, P. L., Ponte, E., Sigel, I., Braun, H., & Powers, D. (2005). *A portrait of advanced placement teachers' practices*. (Research Report). Princeton, N J: Educational Testing Service.
- Pajares, F. (1996). Self-efficacy beliefs and mathematical problem-solving of gifted students. *Contemporary Educational Psychology*, 21(4), 325-344.
- Pajares, F. (1997). Current directions in self-efficacy research. In M. Maehr & P. Pintrich (Eds.), *Advances in motivation and achievement* (Vol. 10, pp. 1-19). Bingley: Emerald Group.
- Pajares, F., & Miller, M. D. (1995). Mathematics self-efficacy and mathematics performances: The need for specificity of assessment. *Journal of Counselling Psychology*, 42(2), 190.
- Pajares, F., & Schunk, D. H. (2002). Self and self-belief in psychology and education: A historical perspective. In J. Aronson (Ed.), *Improving Academic Achievement* (pp.3-21). New York, NY: Academic Press.
- Parker, D. (2006). Grade 10-12 Mathematics curriculum reform in South Africa: A textual analysis of new national curriculum statements. *African Journal of Research in Mathematics, Science and Technology Education*, 10(2), 59-73.

Parker, W., Mosborg, S., Bransford, J., Vye, N., Wilkerson, J., & Abbott, R. (2011).

Rethinking advanced high school coursework: Tackling the depth/breadth tension in the AP US government and politics course. *Journal of Curriculum Studies*, 43(4), 533-559. doi:10.1080/00220272.2011.584561 .

Pascarella, E. T. & Terenzini P. T. (2005), *How college affects students (Vol 2): A third decade of research*. San Francisco, CA: Jossey-Bass.

Patten, M. L., Bruce, R., (2009). *Understanding research methods: An overview of the essentials*. Glendale, CA: Pyrczak Publications.

Patton, M. Q. (1990). *Qualitative evaluation and research methods* (2nd ed). Thousand Oaks, CA: Sage Publications.

Persson, R. S. (2014). The needs of the highly able and the needs of society: A Multidisciplinary Analysis of Talent Differentiation and its significance to gifted education and issues of societal inequality, *Roeper Review*, 36(1), 43-59, Doi: 10.1080/02783193.2013.856830

Peterson, C. (2000). The future of optimism. *American Psychologist*, 55(1), 44.

Pintrich, P. R., & De Groot, E. V. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology*, 82(1), 33.

Pintrich, P. R., & Garcia, T. (1994). Self-regulated learning in college students: Knowledge, strategies, and motivation. In *Student Motivation, Cognition, and*

- Learning: Essays in Honor of Wilbert J. McKeachie* (pp. 113-133). New York, NY: Erlbaum.
- Pintrich, P. R., & Schunk, D. H. (1996). *Motivation in education: Theory, research, and applications*. Upper Saddle River, NJ: Pearson/Merrill Prentice Hall.
- Plowright, D. (2011). *Using mixed methods: Framework for an integrated methodology*. London: Sage Publications.
- Plowright, D. (2013). To what extent do postgraduate students understand the principles of mixed methods in educational research? *International Journal of Multiple Research Approaches*, 7(1), 66-82.
- Poropat, A. E. (2009). A meta-analysis of the five-factor model of personality and academic performance. *Psychological Bulletin*, 135(2), 322-338.
doi:10.1037/a0014996
- Potgieter, M., Davidowitz, B., & Venter, E. (2008). Assessment of preparedness of first-year chemistry students: Development and application of an instrument for diagnostic and placement purposes. *African Journal of Research in Mathematics, Science and Technology Education*, 12 (1), 1-17.
- Pressey, S. L. (1949). *Educational acceleration: Appraisals and basic problems*. (Research Report). Columbus, OH: Ohio State University.
- Putnam, H. (1987). *The many faces of Realism*. Cambridge: Cambridge University Press.

- Rankin, N., Schöer, V., Sebastiao, C., & Van Walbeek, C. (2012). Predictors of academic performance: National Senior Certificate versus National Benchmark Test. *South African Journal of Higher Education*, 26(3), 564-585.
- Renzulli, J. S. (1983). Guiding the gifted in the pursuit of real problems: The transformed role of the teacher. *The Journal of Creative Behaviour*, 17(1), 49-59.
- Richardson, M., Abraham, C., & Bond, R. (2012). Psychological correlates of University students' academic performance: A systematic review and meta-analysis. *Psychological Bulletin*, 138(2), 353-387. doi:10.1037/a0026838
- Robbins, S. B., Lauver, K., Le, H., Davis, D., Langley, R., & Carlstrom, A. (2004). Do psychosocial and study skill factors predict college outcomes? A meta-analysis. *Psychological Bulletin*, 130(2), 261.
- Rogan, J. M. (2007). An uncertain harvest: A case study of implementation of innovation. *Journal of Curriculum Studies*, 39(1), 97-121.
doi:10.1080/00220270500532448
- Rose, H., & Betts, J. R. (2001). *Math matters: The links between high school curriculum, college graduation, and earnings*. San Francisco, CA: Public Policy Institute of California.
- Rothschild, E. (1999). Four decades of the advanced placement program. *The History Teacher*, 32(2, Special Issue: Advanced Placement), 175-206.
- RSA (1996). The Bill of Rights of the Constitution of the Republic of South Africa. *Government Gazette*. (No. 17678). Pretoria: Government Printer.

- Sadler, P. M., & Tai, R. H. (2007). Advanced placement exam scores as a predictor of performance in introductory college biology, chemistry and physics courses. *Science Educator*, 16(2), 1-19.
- Sadler, P. M., Sonnert, G., Tai, R. H., & Klopfenstein, K. (2010). *AP: A critical examination of the advanced placement program*. Cambridge: Harvard Education Press.
- SAIRR (South African Institute of Race Relations). (2012). *SA Education Survey*. Pretoria: South African Institute of Race Relations.
- Sayed, Y., Kanjee, A., & Nkomo, M. (2013). *The search for quality education in post-apartheid South Africa interventions to improve learning and teaching*. Cape Town: Human Sciences Research Council Press.
- Schavelson, R., & Bolus, R. (1982). Self-concept: The interplay of theory and models. *Journal of Educational Psychology*, 74, 3-17.
- Schneider, J. (2009). Privilege, equity, and the advanced placement program: Tug of war. *Journal of Curriculum Studies*, 41(6), 813-831.
doi:10.1080/00220270802713613
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334-370). New York, NY: MacMillan.

- Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education*, 4(1), 1-94.
- Schoenfeld, A. H. (2010). [Review of the book Theories of mathematics education: Seeking new frontiers. (Springer series: Advances in mathematics education) by B. Sriraman and L. English]. *ZDM Mathematics Education*, 42(5), 503-506.
- Schöer, V., Ntuli, M., Rankin, N., Sebastiao, C., & Hunt, K. (2010). A blurred signal? The usefulness of national senior certificate (NSC) mathematics marks as predictors of academic performance at university level. *Perspectives in Education*, 28(2), 9-18.
- Schunk, D. H. (1984). Self-efficacy perspective on achievement behaviour. *Educational Psychologist*, 19(1), 48-58.
- Schunk, D. H. (1991). Self-efficacy and academic motivation. *Educational Psychologist*, 26(3-4), 207-231.
- Schunk, D. H. (1996). *Learning theories*. Boston, MA: Pearson.
- Schunk, D. H., & Hanson, A. R. (1985). Peer models: Influence on children's self-efficacy and achievement. *Journal of Educational Psychology*, 77(3), 313.
- Schwab, K. (2013). *The Global Competitiveness Report 2013–2014*. Geneva: World Economic Forum.
- Scott, I., Ndebele, N., Badsha, N., Figaji, B., Gevers, W., & Pityana, B. (2013). *A proposal for undergraduate curriculum reform in South Africa: The case for a flexible curriculum structure*. Pretoria: Council on Higher Education.

- Scott, I., Yeld, N., & Hendry, J. (2007). *Higher education monitor: A case for improving teaching and learning in South African higher education*. Pretoria: Council on Higher Education.
- Scott, T. P., Tolson, H., & Yi-Hsuan Lee. (2010). Assessment of advanced placement participation and university academic success in the first semester: Controlling for selected high school academic abilities. *Journal of College Admission*, (208), 26-30.
- Seidman, A. (2005). *College student retention: Formula for student success*. Santa Barbara, CA: Greenwood.
- Selamolela, N. & Masonso, T., (2013). *CHE reports' analysis unsound and narrow-SASCO*; 23 Augustus 2013. Retrieved from website www.politicsweb.co.za
- Shaw, E. J., Marini, J. P., & Mattern, K. D. (2013). Exploring the utility of advanced placement participation and performance in college admission decisions. *Educational and Psychological Measurement*, 73(2), 229-253. doi:10.1177/0013164412454291
- Shiu, A., Kettler, T., & Johnsen, S. K. (2009). Social effects of Hispanic students enrolled in an AP class in middle school. *Journal of Advanced Academics*, 21(1), 58-82.
- Shuell, T. J. (1986). Cognitive conceptions of learning. *Review of Educational Research*, 56(4), 411-436.
- Simkins, C. (2013). *Performance in the South African educational system: What do we know*. Johannesburg: Center for Development and Enterprise.

- Simon, M. A. (2009). Amidst multiple theories of learning in mathematics education. *Journal for Research in Mathematics Education*, 40(5), 477-490.
- Skinner, B. F. (1953). *Science and human behaviour*. Cambridge, MA: Simon and Schuster.
- Skovsmose, O. (2000). Aporism and critical mathematics education. *For the Learning of Mathematics*, 20(1), 2-8.
- Smith, J. P., & Naylor, R. A. (2001). Dropping out of university: A statistical analysis of the probability of withdrawal for UK university students. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 164(2), 389-405.
- Snow, R. E. (1987). Aptitude complexes. *Aptitude, Learning and Instruction*, 3, 11-34.
- Southern, W. T., Jones, E., & Stanley, J. (1993). Acceleration and enrichment: The context and development of program options. In K. H. Heller, F.J. Mönks, R. Sternberg, & R. Subotnik, *International Handbook of Research and Development of Giftedness and Talent* (pp. 387-409).
- Spaull, N. (2013). *South Africa's education crisis: The quality of education in South Africa 1994–2011*. (Centre for Enterprise and Development Report). Johannesburg: Centre for Development and Enterprise.
- Sproule, S. (n. d.). *Qualification generation process report Advanced Programme Mathematics*. Johannesburg: IEB.

- Squires, D. A. (2008). *Curriculum alignment: Research-based strategies for increasing student achievement*. Thousand Oaks, CA: Sage Publishers.
- Sriraman, B. (2002). How do mathematically gifted students abstract and generalize mathematical concepts? *NAGC, 2002 Research Briefs*, 16, 83-87.
- Sriraman, B., & English, L. (2010). Surveying theories and philosophies of mathematics education. In B. Sriraman & L. English, *Theories of Mathematics Education: Seeking new frontiers (Advances in Mathematics Education)* (pp.7-32). Dordrecht: Springer.
- Stankov, L., & Lee, J. (2014). Quest for the best non-cognitive predictor of academic achievement. *Educational Psychology*, 34(1), 1-8.
- Stankov, L., Morony, S., & Lee, Y. P. (2014). Confidence: The best non-cognitive predictor of academic achievement? *Educational Psychology*, 34(1), 9-28.
- Stankov, L. (2014). Non-cognitive predictors of academic achievement and intelligence: An important role of self-confidence. *Personality and Individual Differences*, 60 (Supplement), S1-S82.
- Stankov, L., Lee, J., Luo, W., & Hogan, D. J. (2012). Confidence: A better predictor of academic achievement than self-efficacy, self-concept and anxiety? *Learning and Individual Differences*, 22(6), 747-758.
- Statistics South Africa. (2013). *Statistics South Africa 2013*. Pretoria: Statistics South Africa.

- StatSoft Inc. (2013). *Electronic Statistics Textbook*. Tulsa, OK: StatSoft. Retrieved from <http://www.statsoft.com/textbook/>
- Steffe, L. P., & Gale, J. E. (1995). *Constructivism in education*. Hillsdale, NJ: Erlbaum
- Steffe, L. P., Nesher, P., Cobb, P., Sriraman, B., & Greer, B. (2013). *Theories of mathematical learning*. New York, NY: Routledge.
- Sternberg, R. J., & Davidson, J. E. (2005). *Conceptions of giftedness*. Cambridge: Cambridge University Press.
- Stinson, D. W. (2004). Mathematics as “gate-keeper” (?): Three theoretical perspectives that aim toward empowering all children with a key to the gate. *The Mathematics Educator*, 14(1), 8-18.
- Strydom, F., Kuh, G., & Mentz, M. (2010). Enhancing success in South Africa's higher education: Measuring student engagement. *Acta Academica*, 42(1), 259-278.
- Sue, V. M., & Ritter, L. A. (2012). *Conducting online surveys*. Thousand Oaks, CA: Sage Publications.
- Swail, W. S. (2004). *The art of student retention: A handbook for practitioners and administrators*. Austin, TX: Educational Policy Institute.
- Tall, D. (1991). *Advanced mathematical thinking*. New York, NY: Springer.

- Taylor, C., Kokot, S., & Heller, K. (2000). The status of gifted child education in Africa. In K. A. Heller, F. J. Mönks, R. Subotnik, R. J. Sternberg (Eds.), *International handbook of Giftedness and Talent*. Oxford: Elsevier Science.
- Taylor, R. (1990). Interpretation of the correlation coefficient: A basic review. *Journal of Diagnostic Medical Sonography*, 6(1), 35-39.
- Teddlie, C., & Tashakkori, A. (2003). Major issues and controversies in the use of mixed methods in the social and behavioural sciences. In A. Tashakkori & C. Teddlie, *Handbook of Mixed Methods in Social & Behavioural Research* (pp. 3-50). Thousand Oaks, CA: Sage Publications.
- Tinto, V. (1975). Dropout from higher education: A theoretical synthesis of recent research. *Review of Educational Research*, 89-125.
- Tinto, V. (1987). *Leaving college: Rethinking the causes and cures of student attrition*. Chicago, ILL: University of Chicago Press.
- Tinto, V. (1999). Taking retention seriously: Rethinking the first year of college. *NACADA Journal*, 19(2), 5-9.
- Tinto, V. (2006). Research and practice of student retention: What next? *Journal of College Student Retention: Research, Theory and Practice*, 8(1), 1-19.
- Trümpelmann, M. H., & Nel, P. R. T. (1991). *The Joint Matriculation Board: Seventy five years, achievement in perspective*. Pretoria: University of South Africa.

- Tu, W. (1998). Confucius and Confucianism. In W. Slote & G. D. DeVos, *Confucianism and the Family* (pp. 3-36). Albany, NY: State University of New York Press.
- UK NARIC (2014): *Benchmarking Analysis: The National Senior Certificate (Republic of South Africa)*. Retrieved from <http://www.naric.org.uk/naric/research%20and%20consultancy/qualification%20benchmarking%20case%20studies.aspx>
- UNESCO. (1994). *The Salamanca statement and framework for action on special needs education: Adopted by the world conference on special needs education;access and quality*. Salamanca, Spain, 7-10 June 1994. Paris: UNESCO.
- Van der Berg, S. (2007). Apartheid's enduring legacy: Inequalities in education. *Journal of African Economies*, 16(5), 849-880.
- Van der Berg, S. (2008). How effective are poor schools? Poverty and educational outcomes in South Africa. *Studies in Educational Evaluation*, 34(3), 145-154.
- Van der Merwe, H. (1996). The research process: Problem statement and research design. *Effective Research in the Social Sciences*. Pretoria: Van Schaik.
- Van der Westhuizen, C., & Maree, J. G. (2006). Some thoughts on the training of teachers of gifted learners. *Gifted Education International*, 21(2-3), 201-217.
- Vaughn, E. S. (2010). Reform in an urban school district: The role of PSAT results in promoting advanced placement course-taking. *Education and Urban Society*, 42(4), 394-406.

- Vithal, R., Adler, J., & Keitel, C. (2005). *Researching mathematics education in south africa: Perspectives, practices and possibilities*. Cape Town: Human Sciences Research Council Press.
- Vithal, R., & Alant, B. (2005). Speaking the Curriculum: Learner Voices and Silences- Challenges for mathematics and science education in the twenty-first century. *Perspectives in Education*, 23(5), vii. Editorial Part 1.
- Volmink, J. (1994). Mathematics by all. *Cultural perspectives on the Mathematics classroom*, 51-67. New York, NY: Springer.
- Von Glasersfeld, E. (1995). *Radical constructivism: A way of knowing and learning*. *Studies in Mathematics Education Series: 6*. Bristol, PA: Falmer Press
- Vroom, V. H. (1964). *Work and motivation*. New York, NY: John Wiley & Sons.
- Wakelyn, D. (2009). *Raising rigor, getting results: Lessons learned from AP expansion*. (Project for NGA Center for Best Practices). Washington, DC: NGA Center.
- Watters, J. J., & Diezmann, C. M. (1997). Optimising activities to meet the needs of young children gifted in mathematics and science. In P. Rillero & J Allison, *Creative Childhood Experiences in Mathematics and Science. Projects, Activity Series, and Centres for Early Childhood* (pp. 143-170). Columbus, OH: ERIC/CSMEE
- Whiting, G., & Ford, D. (2006). Under-representation of diverse students in gifted education: Recommendations for non-discriminatory assessment (part 2). *Gifted Education Press Quarterly*, 20(3), 6-10.

- Wiley, E. (1989). Advanced placement program could spell benefits for minorities. *Black Issues in Higher Education*, 5(21), 1-15.
- Willingham, W. W., & Morris, M. M. (1986). *Four years later: A longitudinal study of advanced placement students in college*. (College Board Report no 86-2). Princeton, NJ: College Board
- Wolmarans, N., Smit, R., Collier-Reed, B., & Leather, H. (2010). *Addressing concerns with the NSC: An analysis of first –year student performance in Mathematics and Physics*. Paper presented at the 18th Conference of the Southern African Association for Research in Mathematics, Science and Technology Education (pp. 274 - 284). Kwazulu- Natal, South Africa.
- Wood, R. E., & Locke, E. A. (1987). The relation of self-efficacy and grade goals to academic performance. *Educational and Psychological Measurement*, 47(4), 1013-1024.
- Xolo, S. (2007). Developing the potential of the gifted disadvantaged in South Africa. *Gifted Education International*, 23(2), 201-206. doi:10.1177/026142940702300211
- Yeld N. (2012, Desember 12). SA must work to dig basic education out of deep trouble. *Business Day Live, African Perspectives*, Retrieved from <http://www.bdlive.com>
- Yeld, N. (2009). *National Benchmark Tests Project as a national service to higher education*. (Summary Report June 2009). Pretoria: Higher Education South Africa.

- Yilmaz, K. (2011). The cognitive perspective on learning: Its theoretical underpinnings and implications for classroom practices. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas*, 84(5), 204-212.
- Yoon, S. Y., & Gentry, M. (2009). Racial and ethnic representation in gifted Programs: Current status of and implications for gifted Asian American students. *Gifted Child Quarterly*, 53(4), 121-136.
- Zimmerman, B. J., & Schunk, D. H. (2008). An essential dimension of self-regulated learning. In D. H. Schunck & B. J. Zimmerman (Eds.), *Motivation and Self-Regulated Learning: Theory, Research, and Applications* (pp. 1-30). New York, NY: Erlbaum.

APPENDIX A: LETTER OF ETHICAL CLEARANCE



UNIVERSITEIT STELLENBOSCH UNIVERSITY
jou kennisvennoot • your knowledge partner

Approval Notice New Application

30-Sep-2013
DU PLESSIS, Hester Maria

Proposal #: DESC_DuPlessishester2013

Title: Evaluating the effectiveness of Advanced Programme Mathematics (APM) in preparing learners for university Mathematics.

Dear Mrs Hester DU PLESSIS,

Your DESC approved **New Application** received on **19-Aug-2013**, was reviewed by members of the **Research Ethics Committee: Human Research (Humanities)** via Expedited review procedures on **27-Sep-2013** and was approved.

Please note the following information about your approved research proposal:

Proposal Approval Period: 27-Sep-2013 -26-Sep-2014

Please take note of the general Investigator Responsibilities attached to this letter. You may commence with your research after complying fully with these guidelines.

Please remember to use your **proposal number** (**DESC_DuPlessishester2013**) on any documents or correspondence with the REC concerning your research proposal.

Please note that the REC has the prerogative and authority to ask further questions, seek additional information, require further modifications, or monitor the conduct of your research and the consent process.

Also note that a progress report should be submitted to the Committee before the approval period has expired if a continuation is required. The Committee will then consider the continuation of the project for a further year (if necessary).

This committee abides by the ethical norms and principles for research, established by the Declaration of Helsinki and the Guidelines for Ethical Research: Principles Structures and Processes 2004 (Department of Health). Annually a number of projects may be selected randomly for an external audit.

National Health Research Ethics Committee (NHREC) registration number REC-050411-032.

We wish you the best as you conduct your research.

If you have any questions or need further help, please contact the REC office at 0218839027.

Included Documents:

Permission letter US
Permission letter
Interview schedule
Informed consent form
Research proposal
DESC form

Sincerely,

Susara Oberholzer
REC Coordinator
Research Ethics Committee: Human Research (Humanities)

APPENDIX B: INSTITUTIONAL PERMISSION



UNIVERSITEIT • STELLENBOSCH • UNIVERSITY
jou kennisvennoot • your knowledge partner

10 June 2013

Ms Hester M. du Plessis
Faculty of Education
Stellenbosch University

Dear Ms du Plessis

Re: *Evaluating the effectiveness of Advanced Programme Mathematics (APM) in preparing learners for university Mathematics*

In principle this research project is approved. However, direct access to the University's databases cannot be given to the researcher. The researcher must contact Mr Jan du Toit, Assistant Registrar and Head of Student Information System Support (SISS), for assistance and support with accessing student records.

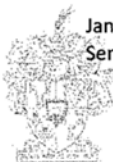
Institutional permission is granted on the following conditions:

- the researcher must obtain ethical clearance from the SU Research Ethics Committee,
- the researcher must obtain the participants' full informed consent,
- participation is voluntary,
- participants may withdraw their participation at any time, and without consequence,
- data must be collected in a way that ensures the anonymity of all participants,
- individuals may not be identified in the results of the study,
- data that is collected may only be used for the purpose of this study,
- data that is collected must be destroyed on completion of this study,
- the privacy of individuals must be respected and protected.

The researcher must act in accordance with SU's principles of research ethics and scientific integrity as stipulated in the *Framework Policy for the Assurance and Promotion of Ethically Accountable Research at Stellenbosch University*.

Best wishes,

Jan Botha
Senior Director Institutional Research and Planning Division



Afdeling Institusionele Navorsing en Beplanning • Institutional Research and Planning Division
Privaatsak/Private Bag X1 • Stellenbosch • 7602 • Suid-Afrika/South Africa
Tel. +27 21 808 3967 • Faks/Fax +27 21 808 4533



APPENDIX C: PERMISSION FROM IEB

From: Helen Sidiropoulos
Sent: 04/04/13 08:42 AM
To: este@africamail.com
Subject: AP Maths & Masters Studies

Dear Este

The IEB will allow permission for the following :

- 1.Data on the results of AP Maths per module/question , uptake , and averages. No individual learner results will be made available. (I have attached the IDC report which gives the results per module/question and a presentation which explains the status of AP Maths and the results and uptake since 2008).
- 2.Use of the AP Maths Curriculum .

As for how the subject came about : it was an initiative that started after Advanced Mathematics was removed from the qualification. I have attached the original documents (1) and (2). You will see on page 4 of document (1) the consultation process and people involved. The team leader was Dr Stephen Sproule.

We would be very interested in the research findings , and ask that these be made available to us.

Regards

Dr Helen Sidiropoulos
Assessment Specialist
5 Anerley Road, Parktown, 2193
PO Box 875, Highlands North 2037

Tel : +27 (011) 483 9720

Fax: 0865294888

E-mail: sidiropoulos@ieb.co.za

Website: www.ieb.co.za

Confidentiality Note



APPENDIX D: QUESTIONNAIRE

QUESTIONNAIRE

Invitation to participate

MATHS SURVEY

Dear Student

Thank you that you are willing to complete this survey. It consists of 12 questions on your experience of the transition from high school Mathematics to first year university Mathematics. It will take no longer than 10 minutes to complete.

All responses will be treated strictly confidential and will be used for research purposes only. It will be part of a thesis for my Masters' study.

Your input will contribute towards guiding future parents and learners in their choice of high school subjects. It will give curriculum planners insight into the skills required for successful transition from school to university Mathematics.

Thank you so much for your time!

Hester du Plessis

For more information on the research project, open this attachment:

Questionnaire

MATHS SURVEY



Question 1

Did you take the courses MATHEMATICS 114 and/or MATHEMATICS 144 or MATHEMATICS 115 and/or MATHEMATICS 145 for the first time in 2013?

- ☐ Yes
- ☐ No, I am repeating a first semester course.
- ☐ No, I am repeating a second semester course.
- ☐ No, I am repeating a first as well as a second semester course.



Question 2

Please rate your experience of the initial transition from School Mathematics to University Mathematics on a scale from 1 to 5. Choice 1 means that you struggled a lot while choice 5 means that you had no problems.

	1	2	3	4	5
My experience of the transition from school to university Mathematics	option 1	option 2	option 3	option 4	option 5



Question 3

In which category does your Grade 12 final mark for Mathematics fall?



Question 4

Did you take the subject ADVANCED PROGRAMME MATHEMATICS (APM) during your Grade 12 year in high school?

☒ Yes ☐ No

If you selected No at the previous question, you do not have to answer any of the following questions. Please just submit at the end. Thank you for your time.

Question 5

In what ways did the fact that you took ADVANCED PROGRAMME MATHEMATICS at school help you in the courses MATHEMATICS 114 and MATHEMATICS 144 (or MATHEMATICS 115 and MATHEMATICS 145)? Please rate your level of agreement by ticking one box in each row.

	strongly disagree	disagree	neither agree nor disagree	agree	strongly agree
Having had APM gave me confidence in my mathematical abilities	option 1 <input type="radio"/>	option 2 <input type="radio"/>	option 3 <input type="radio"/>	option 4 <input type="radio"/>	option 5 <input type="radio"/>
Having had APM helped me with my time management	option 1 <input type="radio"/>	option 2 <input type="radio"/>	option 3 <input type="radio"/>	option 4 <input type="radio"/>	option 5 <input type="radio"/>
Having had APM helped me to handle the large workload	option 1 <input type="radio"/>	option 2 <input type="radio"/>	option 3 <input type="radio"/>	option 4 <input type="radio"/>	option 5 <input type="radio"/>
Having had APM made it easier for me to understand the mathematics	option 1 <input type="radio"/>	option 2 <input type="radio"/>	option 3 <input type="radio"/>	option 4 <input type="radio"/>	option 5 <input type="radio"/>

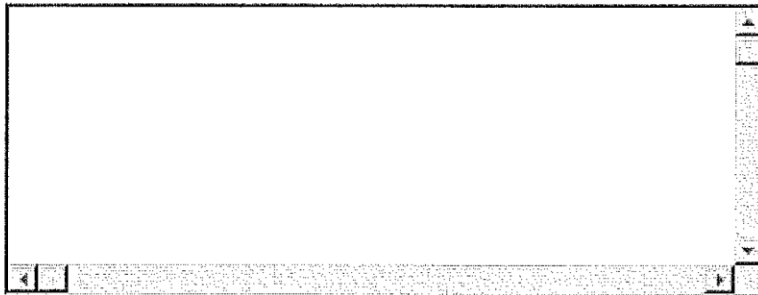
Question 6

Did the fact that you had ADVANCED PROGRAMME MATHEMATICS in high school result in you not attending classes in MATHEMATICS 114 and MATHEMATICS 144 (or MATHEMATICS 144 and MATHEMATICS 145)?

- ☐ Never
☐ Sometimes
☒ Quite often

Question 7

In which other ways did the fact that you took APM in high school have a positive or negative influence on your experience of first year Mathematics?



Question 8

In which category does your Grade 12 mark for Advanced Programme Mathematics fall?

- ☐ 90% +
- ☐ 80%-89%
- ☐ 79%-79%
- ☐ 60%-69%
- ☐ Less than 60%

Question 9

Now, after a year of university Mathematics, how would you rate the mathematical knowledge of your APM teacher in Grade 12?

inadequate	basic	adequate	superior
option 1	option 2	option 3	option 4

Question 10

Where did you receive your high school training?

Choose...

Question11

Would you recommend APM to a friend in high school?

- ☐ Yes
 ☐ No
 ☐ No answer

If you selected No in the previous question, please specify why.

APPENDIX E: EXAMINATION PAPERS

NSC MATHEMATICS 2012 PAPERS 1, 2, and 3: <http://www.education.gov.za/>

ADVANCED PROGRAMME MATHEMATICS (2012)



GRADE 12 EXAMINATION
NOVEMBER 2012

ADVANCED PROGRAMME MATHEMATICS

Time: 3 hours

300 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 20 pages and an Information Booklet of 4 pages (i – iv). An Answer Sheet is also provided for use with Questions 2.3 (a) and 4.1 (a) of Module 1 and this should be handed in with your Answer Book. Please check that your question paper is complete.
 2. This question paper consists of FOUR Modules:
MODULE 1: CALCULUS AND ALGEBRA (200 marks) is compulsory.
Choose **ONE** of the **THREE** Optional Modules:
MODULE 2: STATISTICS (100 marks) OR
MODULE 3: FINANCE AND MODELLING (100 marks) OR
MODULE 4: MATRICES AND GRAPH THEORY (100 marks)
 3. Non-programmable and non-graphical calculators may be used.
 4. All necessary calculations must be clearly shown and writing should be legible.
 5. Diagrams have not been drawn to scale.
 6. If applicable, calculations should be done using radians and answers should be given in radians.
 7. Round off your answers as indicated in each question.
-

MODULE 1 CALCULUS AND ALGEBRA

QUESTION 1

Prove by induction that $9^n - 8n - 1$ is divisible by 8 for all natural values of n , with $n > 1$.

[14]

QUESTION 2

2.1 Solve for x without using a calculator:

$$\log x = \log 2 + \log(x+5) - \log(x-1). \quad (6)$$

2.2 Given $f(x) = \frac{1}{1+e^x}$

(a) Determine whether $f(x)$ is increasing or decreasing. (3)

(b) What is the range of $f(x)$? (4)

(c) Find an expression for the inverse function $f^{-1}(x)$. (4)

2.3 Given: $f(x) = \begin{cases} x & \text{if } x \geq 1 \\ -x & \text{if } x < 1 \end{cases}$ and $g(x) = \ln x$

(a) Sketch the graph of f on the Answer Sheet provided. (4)

(b) Determine the domain of $g(f(x))$. (4)

[25]

QUESTION 3

3.1 It is given that $x = a + bi$ satisfies the equation: $(2+i)(x+3i) = 8i+6$.
Find the values of a and b . (7)

3.2 Find the value of k if $x^3 - 5x^2 + kx - 13 = 0$ and $x = 2 + 3i$ is a root of the equation. (8)

[15]

QUESTION 4

4.1 Jamie is playing on his calculator and he notices that $\sin\left|\frac{\pi}{4}\right| = \left|\sin\frac{\pi}{4}\right|$. He tries this for π , $\frac{\pi}{2}$ and $\frac{4}{5}\pi$ and now he believes that he has discovered a new trigonometry identity: $\sin|x| = |\sin x|$.

- (a) Draw the graphs $g(x) = \sin|x|$ and $h(x) = |\sin x|$ on the interval $0 \leq x \leq 2\pi$. Use the Answer Sheet provided. (8)
- (b) Explain Jamie's mistake in thinking that $\sin|x|$ is always equal to $|\sin x|$. (2)

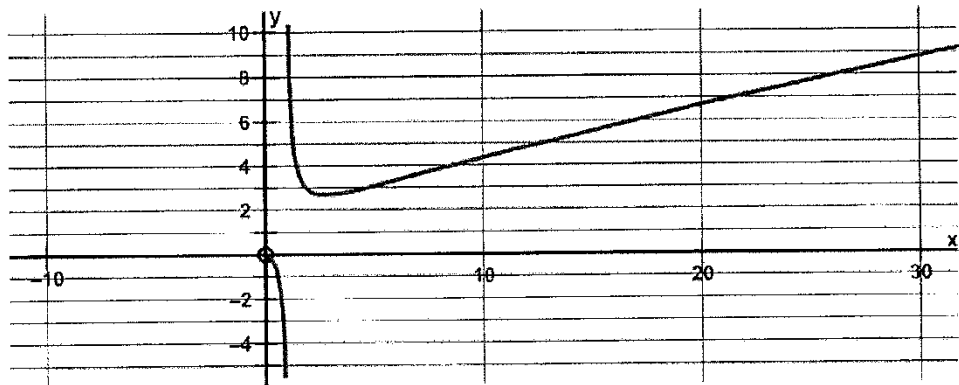
4.2 Given:
$$f(x) = \begin{cases} px & \text{if } 0 < x < \pi \\ -2\cos x & \text{if } \pi \leq x \leq \frac{3\pi}{2} \\ \frac{1}{\pi}\left(x - \frac{5\pi}{2}\right)^2 - 2 & \text{if } x > \frac{3\pi}{2} \end{cases}$$

- (a) (i) Determine the value of p such that f is continuous at $x = \pi$. (You do NOT need to use first principles.) (4)
- (ii) Determine whether f is continuous at $x = \frac{3\pi}{2}$. (5)
- (b) (i) Either by means of a sketch or algebraically, explain why f cannot be differentiable at $x = \pi$ for the value of p found in Question 4.2 (a)(i). (5)
- (ii) Determine $\lim_{x \rightarrow \left(\frac{3\pi}{2}\right)^-} f'(x)$ and $\lim_{x \rightarrow \left(\frac{3\pi}{2}\right)^+} f'(x)$. (6)
- (iii) Is f differentiable at $x = \frac{3\pi}{2}$? Give a reason for your answer. (2)

[32]

QUESTION 5

5.1 Below is the graph of $f(x) = \frac{x}{\ln x}$.



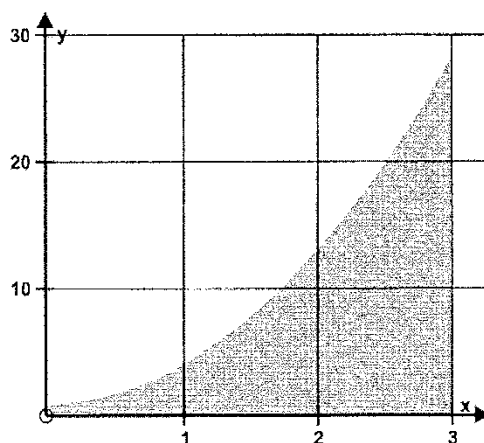
- (a) Explain why there is no graph to the left of the y -axis. (2)
- (b) State the equation of the vertical asymptote of the graph. (2)
- (c) Given that $\frac{d}{dx} \ln x = \frac{1}{x}$, find the coordinates of the turning point of the graph. (8)

5.2 Given the function $f(x) = \frac{2x^2 - 2x + 5}{x + 1}$

- (a) Determine the equation of the oblique asymptote of the graph of f . (6)
 - (b) Does the oblique asymptote intercept the graph? Motivate your answer. (3)
- [21]**

QUESTION 6

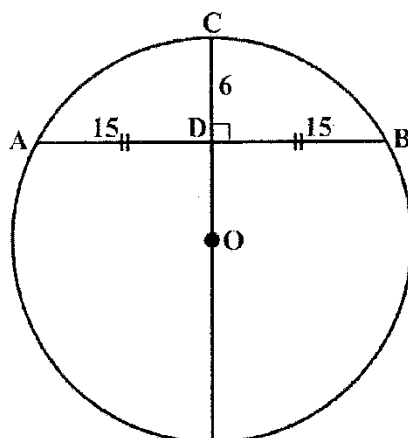
Use a Riemann Sum to determine the area between the curves $f(x) = 3x^2 + 1$ and the x -axis on the interval $x \in [0; 3]$ as indicated on the graph below.



[14]

QUESTION 7

A circle is shown below. CD is perpendicular to AB and when CD is extended it passes through the centre O of the circle. D is the midpoint of AB and the lengths of CD and AB are 6 cm and 30 cm respectively.



- 7.1 Show that the radius of the circle is 21,75 cm. (6)
- 7.2 Determine the arc length ACB, correct to 4 decimal places. (5)
- 7.3 Calculate the area of the segment ACBD, correct to 4 decimal places. (5)

[16]

QUESTION 8

The equation $x^3 - 3xy^2 + y^3 = 1$ defines y as a function of x .

8.1 Verify that $(2 ; -1)$ is a point on the curve. (2)

8.2 Determine $\frac{dy}{dx}$. (8)

8.3 Hence find the equation of the tangent at $(2 ; -1)$. (5)
[15]

QUESTION 9

Find the following integrals:

9.1 $\int 2x^3 - \sec^2\left(\frac{x}{2}\right) - \frac{1}{\sqrt[3]{x}} dx$ (6)

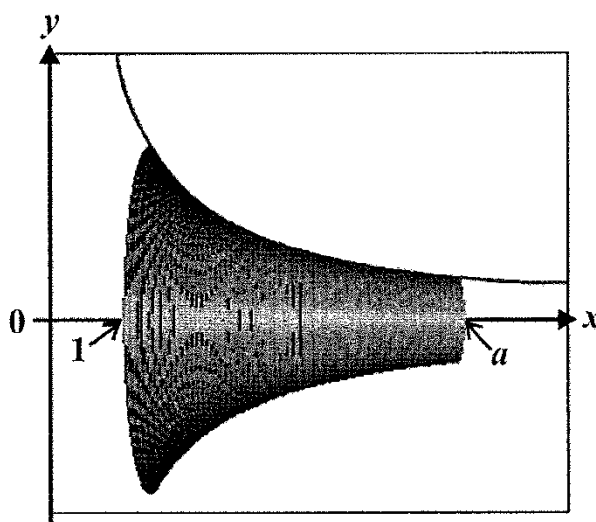
9.2 $\int (\cos^2 3x)(\sin 3x) dx$ (7)

9.3 $\int \cos 4\theta \cdot \sin 5\theta d\theta$ (6)

9.4 $\int y\sqrt{y+3} dy$ (9)
[28]

QUESTION 10

Below is a picture of the graph $y = \frac{1}{x}$. The area bounded by the curve and the lines $x = 1$ and $x = a$, where $a > 1$, has been rotated about the x -axis.



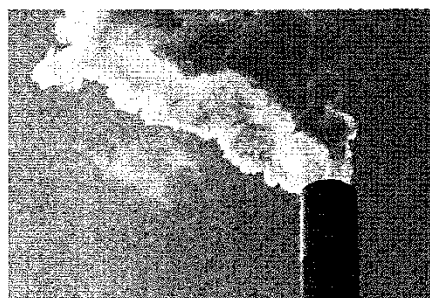
- 10.1 Determine the volume of revolution, in terms of a . (8)
- 10.2 Hence determine the volume of revolution as $a \rightarrow \infty$. (2)
- [10]

QUESTION 11

An industrial chimney deposits pollution (soot) onto the ground. If there are two industrial chimneys (A and B) that are 15 km apart, the concentration of soot on the line joining them, at a distance x from chimney A, is given by:

$$S = \frac{8k}{x^2} + \frac{k}{(15-x)^2}$$

where k is a positive constant.



[<<http://interactive.wxvi.org>>]

Find the point on the line joining the two chimneys where the concentration of soot is a minimum.

[10]

Total for Module 1: 200 marks

MODULE 2 STATISTICS

All answers should be rounded to the fourth decimal place.

QUESTION 1

- 1.1 A factory makes large quantities of coloured sweets, and it is known on average that 20% of the sweets are coloured green. A packet contains 20 sweets (a random sample from the factory). Calculate the probability that exactly seven of the sweets are green. (6)
- 1.2 A Comrades runner owns six different pairs of running shoes. Calculate the number of ways of selecting the following:
- (a) 4 shoes from the 12 (1)
- (b) 2 left shoes and 2 right shoes (3)
- (c) 2 pairs of non-matching shoes (Note: a pair is a left and right shoe) (4)
- 1.3 Find the number of ways that 9 people can be arranged into three groups of three. (5)
- 1.4 A batch of 20 integrated circuit chips contains 20% which are defective. A sample of 10 is drawn at random. What is the probability that at least one of the chips will be defective? (7)
- [26]

QUESTION 2

Scores on an IQ test are normally distributed with a mean of 100 and a standard deviation of 15.

- 2.1 It is said that to have superior intelligence, your IQ score must fall within 120 and 129. What percentage of the world's population fall in this category? (10)
- 2.2 2,2% of the population fall in the lowest category. What IQ score is the maximum score that would put someone in the lowest category? (6)
- [16]

QUESTION 3

Ten learners wrote end-of-term examination papers in AP English and French. Their marks, x for AP English and y for French, are summarised as follows:

$$\sum x = 578 \quad \sum y = 607 \quad \sum x^2 = 38378$$

$$\sum y^2 = 41221 \quad \sum xy = 39070$$

- 3.1 Calculate the mean mark for AP English and French. (2)
- 3.2 Hence, calculate the equation of the regression line y on x . (6)
- 3.3 Calculate the correlation coefficient between AP English and French. The formula to calculate the correlation coefficient is as follows:

$$r = \frac{n \sum xy - (\sum x)(\sum y)}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}} \quad (3)$$

Two other students, Allie and Danny, were each absent from one examination paper. Allie scored 85 in AP English but was absent for French, and Danny scored 65 in French but was absent for AP English.

- 3.4 State for which learner it would be appropriate to use the regression line calculated above to estimate a mark for the examination paper for which the learner was absent. Calculate the estimated mark. (3)
- 3.5 Comment briefly on the reliability of this estimated mark. (2)

[16]

QUESTION 4

- 4.1 Professor Dayle Vadi claims: "Despite increased law enforcement efforts, rhino poaching has been **increasing by an average of at least 10 rhino per province per year** in South Africa."

The table below is a breakdown of the number of rhinos poached per province for five different regions for the sample years 2010 and 2011.

Province	2010	2011
Kruger National Park	146	252
North West	57	22
Limpopo	52	73
Kwa-Zulu Natal	38	34
Mpumalanga	17	31

- (a) Calculate the mean and standard deviation (to one decimal place) of rhinos poached per province for the years 2010 and 2011. (Treat the Kruger National Park as a separate province.) (4)
- (b) Hence, set up a formal hypothesis test at the 10% level of significance to test Professor Vadi's claim. (Assume the sample comes from a normal distribution.) (10)
- 4.2 A national safety council wants to estimate the proportion of car accidents that involve pedestrians. How large a sample of accident records must be examined to be 94% confident that the estimate does not differ from the true proportion by more than 0,04? The proportion estimated is 0,25. (6)
- [20]

QUESTION 5

A survey was undertaken at Aphane High School to determine the proportion of learners that had Facebook and Twitter accounts. A random sample showed the following:

- If a learner had a Twitter account then they also had a Facebook account.
- If a learner didn't have a Twitter account, then there was a 90% chance they had a Facebook account.
- The probability that they had a Twitter account was 0,36.

- 5.1 Draw a Venn diagram depicting the information given above. (10)
- 5.2 **Calculate:**
- (a) The probability of having a Facebook account. (2)
- (b) The probability of having neither a Facebook account nor a Twitter account. (1)
- [13]

QUESTION 6

Two learners, Alicia and Rae at Faul School obtained 98% and 40% respectively for Mathematics at the end of the year. The group average for that particular grade was 56% with a standard deviation of 18%, which was considered to be too low. The average was adjusted to 60% with a standard deviation of 12%.

- 6.1 Calculate how many standard deviations Alicia and Rae's marks are from the original mean. (2)
 - 6.2 Hence, calculate their new marks once the adjustment has been made. (6)
 - 6.3 Suggest a plausible reason why the standard deviation was lowered. (1)
- [9]**

Total for Module 2: 100 marks

MODULE 3 FINANCE AND MODELLING

QUESTION 1

- 1.1 Ihsaan's friends and family gave him cash for his eighteenth birthday present. He plans to invest this amount, which comes to R5 600, at an annual interest rate of 6,84%, compounded quarterly. Calculate the interest this once-off investment will earn over five years. (4)
- 1.2 Rose deposits money at the end of each quarter in a savings account that also earns an annual interest rate of 6,84%, compounded quarterly. Calculate the value of her equal quarterly deposits if this account accrues to R10 033,38 over a five-year period. (6)
- 1.3 Solly deposits R420 at the end of every quarter in a savings account that also earns an annual interest rate of 6,84%. However, with the bank he has chosen, interest is compounded monthly. Calculate the value to which his investments will accrue over a five-year period. (10)
[20]

QUESTION 2

During the first two years of the lifespan of a new car, depreciation in its value is usually quite high at 12% per annum. During the third year, depreciation is slightly lower at 10,5%. For the fourth, fifth and sixth years, depreciation remains constant.

The annual rate of depreciation over the first 6 years of the lifespan of a car is shown in the table below:

Year	1	2	3	4	5	6
Rate of depreciation	12%	12%	10,5%	$x\%$	$x\%$	$x\%$

Calculate x if the average annual rate of depreciation over the 6 years is 10% per annum. Give your answer as a percentage, correct to two decimal places.

[8]

QUESTION 3

Andile secures a home loan for R850 000 from a bank at an annual interest rate of 8,56%, compounded monthly. He agrees to repay R7 400 per month at the end of every month.

- 3.1 Calculate the expected outstanding balance on the loan immediately after the 80th payment. (8)
- 3.2 In addition to his monthly repayments, Andile on two occasions deposited his annual bonus into this account to offset his home loan. These deposits of R12 000 each coincided with the 17th and 65th payments. Calculate the new outstanding balance on the loan immediately after the 80th payment. (8)
- 3.3 After the 80th payment, Andile calculates that if he lowers his payment to R7 100 per month, he could pay off his bond after a further 160 payments, the last of which will be less than R7 100. Assuming that the outstanding balance after the 80th payment is R675 000, calculate the value of the final payment. (10)
[26]

QUESTION 4

According to Discovery Channel's NatGeo Wild, wild hogs in North America are causing millions of dollars worth of damage to agricultural land every year. Despite their trail of destruction, they are surprisingly difficult to track down. They can feed on almost anything from roots to rubbish, and don't seem to be preyed on by other animals.

A sow (female hog) has three litters a year, each with an average size of four piglets, 60% of whom will be female. The survival rate of the piglets is high at 80%. The average hog lives for about seven years in the wild.

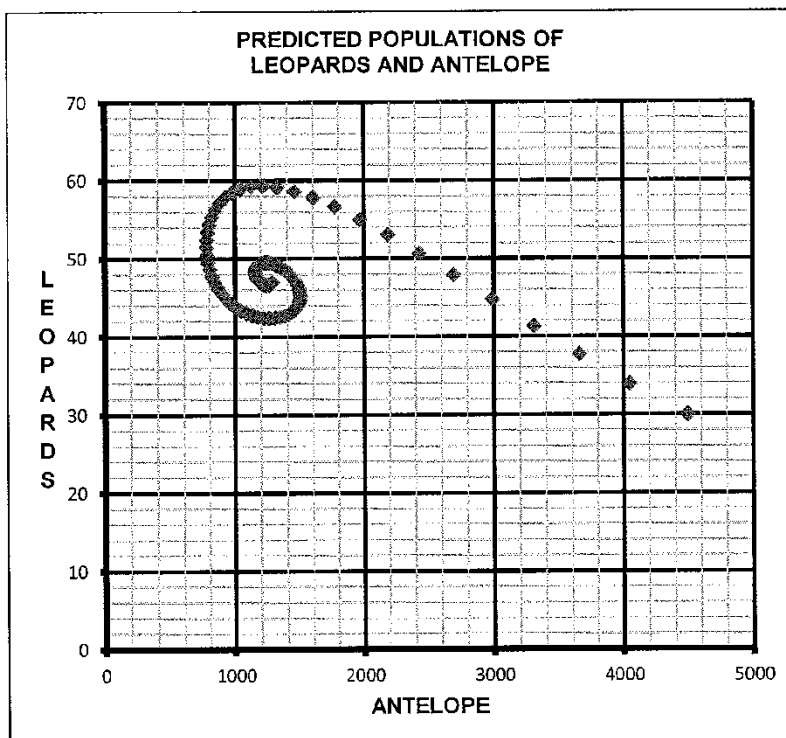
- 4.1 Calculate the annual growth rate of the wild hogs, correct to three decimal places. (6)
- 4.2 Express the growth in the hog population as a recursive formula, using an annual growth rate of 5,6 hogs per annum. (2)
- 4.3 Calculate the hog population after two years, if there are currently 25 hogs on the farm. Use an annual growth rate of 5,6 hogs per annum. (2)
- 4.4 The farmer decides to start exterminating the hogs immediately on a **weekly basis**. Calculate the minimum number of hogs he needs to kill **per week** for the next year, just to keep the number of hogs approximately at their present level. Show suitable calculations to support your answer, rounded to the nearest whole number. (6)

[16]

QUESTION 5

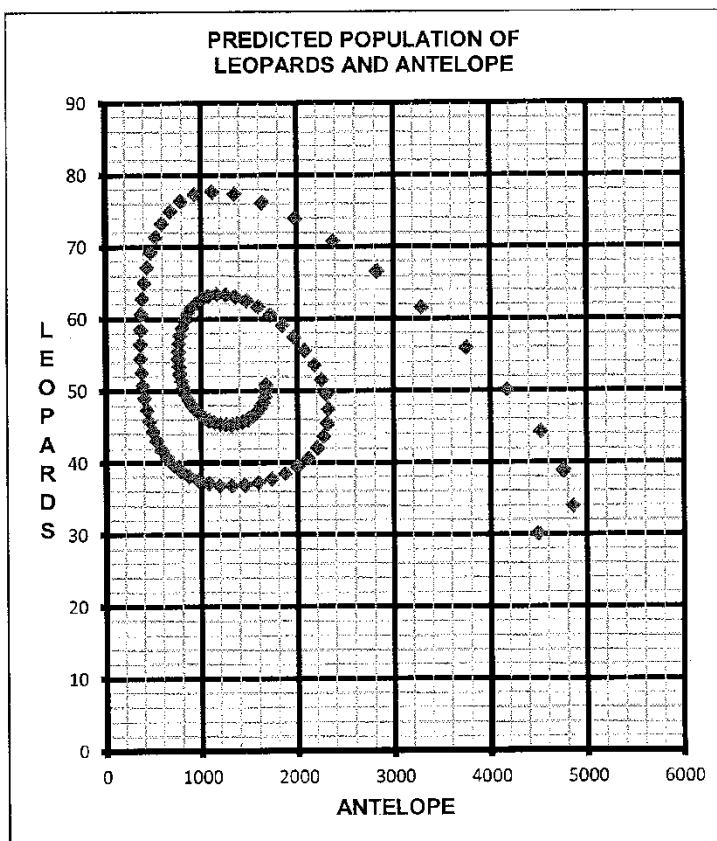
Okapuka is a 10 000 hectare private game farm just 20 km outside Windhoek, Namibia. It is home to a variety of animals, including several antelope species that together have a population of about 4 500. Game rangers have studied leopard tracks, and they estimate that there are 30 leopards in the more mountainous regions of the farm.

- 5.1 The phase plot below represents the predicted populations of leopards and antelope over the next century at Okapuka.



- (a) Read off the equilibrium populations for each species. (2)
- (b) Give the range of the leopard population when the antelope population is increasing for the first time. (2)
- (c) Read off the approximate antelope population when the leopard population is decreasing most rapidly. (2)
- (d) In the first year of this model, leopards killed about 420 antelope. Calculate the value of the parameter b in the predator-prey formula (correct to five decimal places). (5)
- (e) In the first year of this model, four leopard cubs survived. Calculate the value of the parameter f in the predator-prey formula (correct to five decimal places), using $b = 0,003$. (5)

- 5.2 The Okapuka Game Farm is considering buying hundreds of hectares of adjacent land. By doing this it can extend its current borders and will substantially increase the carrying capacity of antelope. The phase plot below represents the new predicted populations of antelope and leopards over the next century.



Compare this phase plot with that in Question 5.1 and then comment on the following aspects:

- (a) the manner in which the initial population of each species changes. (2)
 - (b) the population range of each species. (2)
 - (c) the rate at which the equilibrium point is reached. (2)
- [22]**

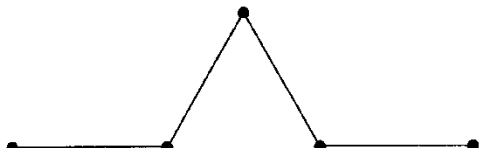
QUESTION 6

Von Koch's Snowflake is a well-known fractal, based on a recursive theme.

Step 1: length 27 units

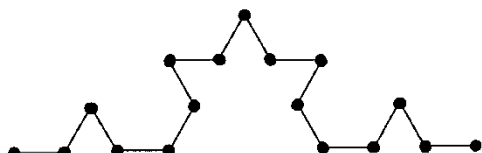


Step 2: length 36 units



This is obtained by the original line segment in Step 1 being trisected. On the middle of the three sections an equilateral triangle has been constructed.

Step 3: length 48 units



This is obtained by each of the four line segments in Step 2 again being trisected. On each of the middle sections of each of the four line segments in Step 2 another equilateral triangle has been constructed.

- 6.1 Write down a first order recursive formula that represents the length of the n^{th} fractal shape. (4)
 - 6.2 Determine which step will be the first where the length exceeds 1 000 units. Also give this length, correct to three decimal places. (4)
- [8]**

Total for Module 3: 100 marks

MODULE 4 MATRICES AND GRAPH THEORY

QUESTION 1

Three matrices $L \begin{pmatrix} 2 & -5 & 0 \\ -1 & 2 & 7 \end{pmatrix}$, $M \begin{pmatrix} 6 & -2 \\ -3 & 1 \end{pmatrix}$, and $N \begin{pmatrix} 5 & 2 \\ -2 & 3 \end{pmatrix}$ are given.

- 1.1 Calculate $2M - 3N$. (4)
 - 1.2 Explain in words why L has no inverse. (2)
 - 1.3 Explain in words why M has no inverse. (2)
 - 1.4 Write down the inverse of N , with integer values for the elements of the matrix. (4)
- [12]**

QUESTION 2

- 2.1 The coordinates of the endpoints of a line segment in a Cartesian plane are $(2 ; -4)$ and $(2 ; 5)$. The line segment is to be transformed by a shear of factor 3, with the x -axis as the invariant line. Calculate the coordinates of the endpoints of the line segment after it has been transformed. (6)
 - 2.2 A figure is to be reflected in the line with equation $y = 3x$. Give the matrix that will effect this transformation. The elements of the matrix must be accurate to one decimal place. (6)
 - 2.3 A figure in a Cartesian plane is to be transformed so that its image has an area three times the original area. Give two different matrices that could be used to effect this transformation. The two matrices must represent different types of transformations. (4)
 - 2.4 The point $P(25 ; -75)$ is to be rotated about the origin, so that its image has coordinates $P'(-45 ; 65)$. Determine the angle of rotation (in degrees and correct to two decimal places). (8)
- [24]**

QUESTION 3

3.1 Consider the matrices $A = \begin{pmatrix} 1 & 2 & -3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -5 & 6 \\ -12 & -10 & -9 \\ -1 & 5 & 8 \end{pmatrix}$

Show through calculation that $AB \neq BA$. (8)

3.2 A matrix is given in the form $\begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}$

Using this matrix, show through calculations that the product of a matrix and its transpose will always be symmetrical about the leading diagonal.

(Remember: the 'transpose' of a matrix is found by interchanging the rows with the columns.)

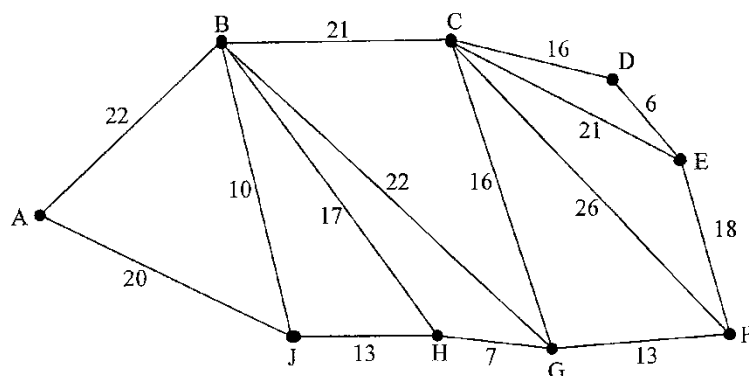
(10)

[18]

QUESTION 4

At the world famous Kirstenbosch Botanical Gardens in Cape Town, a section has been laid out known as the 'Braille Trail'. Visitors are encouraged to experience plant life through senses other than sight by closing their eyes and to be guided through the Trail by holding onto ropes.

The graph below (which is not drawn to scale) represents the 'Braille Trail', with vertex *A* being both the entrance and exit point to the Trail. The edges of the graph correspond to the guide-ropes along the lanes, and the weight of the edges to the length of the ropes in metres.

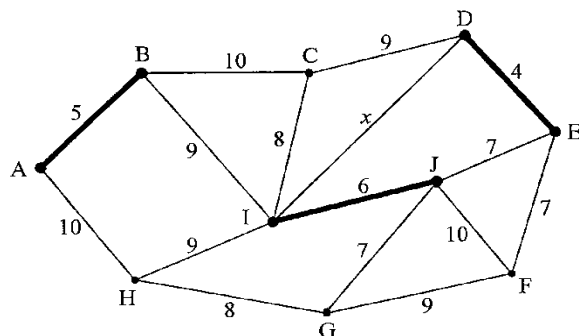


- 4.1 Officials recommend that the minimum distance a visitor walks should be the perimeter of the Trail, namely *ABCDEFHJA*. This would in effect create a Hamiltonian Circuit. Explain in words what makes a circuit 'Hamiltonian'. (2)
- 4.2 To fully experience the Braille Trail, visitors should walk every lane in the Trail. Design an Eulerian Circuit of minimum length for a visitor to fully experience the Trail. Clearly state the actual circuit and the length of the circuit. (12)
- 4.3 Landscapers plan to lay out a new lane that will directly connect *C* and *J*. They decide that, including the new lane *CJ*, a visitor should not have to walk an Eulerian Circuit of more than 300 metres. Calculate what the maximum length of the new lane should be. (10)

[24]

QUESTION 5

In the graph sketched below a student has started finding a spanning tree of minimum length. The first three edges she has chosen have been printed in bold.



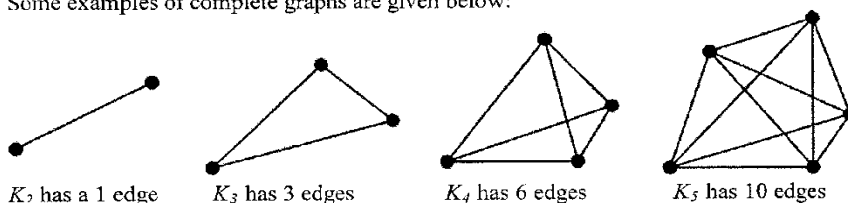
- 5.1 State the number of edges the completed spanning tree will have. (2)
- 5.2 State whether the student is using Kruskal's or Prim's algorithm. Give a reason for your answer. (2)
- 5.3 If DI has to be the next edge selected, state what weight DI must be. (2)
- 5.4 Three edges have a weight of 7. State which of these edges should NOT be selected, once DI has already been selected. Give a reason for your answer. (2)
- 5.5 Complete the spanning tree of minimal length for this graph. Assume that the edges DE, AB, IJ and DI have already been selected. State the length of this minimum spanning tree. (8)

[16]

QUESTION 6

In a **complete** graph every vertex is directly connected to every other vertex. A complete graph is notated by K_n , where n is the number of vertices in the graph.

Some examples of complete graphs are given below:



- 6.1 State the number of edges in the complete graph K_6 . (3)
- 6.2 State for which positive integer values of n the complete graph K_n will inherently contain an Eulerian circuit. (3)

[6]

Total for Module 4: 100 marks

Total: 300 marks

MATHEMATICS 114 (2013)

Voorletters en van
Initials and surname

Studentenr.
Studentno.

Wiskunde / *Mathematics* 114

Eerste Eksamen / *First Exam*

Eerste Vraestel : Kort Vrae / *First Paper: Short Questions*

6 Junie / *June* 2013

Tyd / *Time*: 2h

Volpunte / *Full marks*: 50

Beskikbare punte: / *Available marks*: 52

INSTRUKSIES:

INSTRUCTIONS:

- | | |
|--|--|
| • Hierdie vraestel bestaan uit 4 genummerde bladsye. | <i>This paper consists of 4 numbered pages.</i> |
| • Skryf slegs die antwoorde in die blokkies. | <i>Write only the answers in the boxes.</i> |
| • Gebruik die agterkant van die bladsy vir jou rofwerk. As jy ekstra papier vir rofwerk benodig, mag jy een eksamenantwoordboekie gebruik, maar dit sal nie nagesien word nie. | <i>Use the back of the page for your rough work. If you need extra paper for rough work, you may use one exam answer booklet, but it will not be marked.</i> |
| • Geen boek of geskrif van enige aard mag in die toetslokaal ingebring word nie. | <i>No book or any written material may be brought into the examination room.</i> |
| • Geen deel van hierdie antwoordstel mag uitgeskeur word nie. | <i>No part of this answer sheet may be removed from the examination room.</i> |
| • Geen sakrekenaars toegelaat nie. | <i>No calculators allowed.</i> |
| • Vrae wat met 'n * gemerk is, is meer uitdagend. | <i>Questions marked with a * are more challenging.</i> |

1. Beskou die versameling $A = \{2, 4, 6, 8\}$. Wat- *Consider the set $A = \{2, 4, 6, 8\}$. Which of the fol-*
ter van die volgende bewerings is waar? *lowing assertions are true?*

$$S_1 : \{2, 4, 6\} \subset A, \quad S_2 : \emptyset \in A, \quad S_3 : 4 \subset A, \quad S_4 : \{4\} \subset A$$

 [2]

2. Bereken die volgende limiete, indien hulle bestaan: *Compute the following limits, if they exist:*

(a) $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} =$ [2]

(b) $\lim_{\theta \rightarrow 0} \theta \cdot \operatorname{cosec}(\pi\theta) =$ [2]

(c) $\lim_{x \rightarrow \pi} \frac{\cos x + 1}{x - \pi} =$ [2]

(d) $\lim_{x \rightarrow \infty} \frac{3x^2 + 5x + 1}{2x^2 - 1} =$ [2]

(e) $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 - x + 1}}{x - 1} =$ [2]

(f*) $\lim_{n \rightarrow \infty} \frac{1}{n^{2013}} \sum_{k=1}^n k^{2012} =$ [2]

3. Bereken: / *Compute:* $\sum_{n=0}^{99} \binom{100}{n} 2^n =$ [2]

4. Skryf die volgende komplekse getal in poolvorm met hoofargument: *Write the following complex number in polar form with principal argument:*

$$\sqrt{3} - i =$$
 [2]

5. Skryf neer alle komplekse getalle z wat die volgende vergelyking bevredig: *Write down all complex numbers z satisfying the following equation:*

$$z^3 - 1 = 0$$

$$z =$$
 [2]

6. Indien $\sec y = \frac{5}{4}$ met $y \in [0, \pi/2]$, dan is *If $\sec y = \frac{5}{4}$ with $y \in [0, \pi/2]$, then*

$$\sin 2y =$$
 [2]

7. Bepaal die definisieversameling D_f van die volgende funksie: *Determine the domain of the following function:*

$$f(x) = \frac{\sqrt{x-1}}{\sqrt{27-x^3}}$$

$$D_f =$$
 [2]

8. Bereken die volgende afgeleides:

Compute the following derivatives:

(a) $\frac{d}{dx} \left(8x^5 - \sqrt{x} + \frac{1}{x} \right) =$ [2]

(b) $\frac{d}{dx} \tan^3(\cos x) =$ [2]

(c) $\frac{d^2}{dx^2} x \sin(x) =$ [2]

(d) Indien $x^3 + xy + y^3 = 11$, dan is If $x^3 + xy + y^3 = 11$, then

$\left. \frac{dy}{dx} \right|_{(x,y)=(1,2)} =$ [2]

(e) $\frac{d}{dx} \int_x^\pi \frac{dt}{\sin t + 2} =$ [2]

(f*) $\frac{d^{100}}{dx^{100}} \frac{x-1}{x+1} =$ [2]

9. Gestel f is kontinu op $[1, 4]$ en differensiëerbaar op $(1, 4)$. Gestel $f(4) \leq 4$ en $f'(x) \geq \frac{2}{3}$ vir alle $x \in (1, 4)$. Wat is die grootste wat $f(1)$ kan wees?

Suppose f is continuous on $[1, 4]$ and differentiable on $(1, 4)$. Suppose $f(4) \leq 4$ and $f'(x) \geq \frac{2}{3}$ for all $x \in (1, 4)$. What is the largest that $f(1)$ could be?

[2]

10. Skryf neer die vergelyking van die skuins asimptoot van die grafiek van die volgende funksie:

Write down the equation of the slant asymptote of the graph of the following function:

$f(x) = \frac{2x^3 + 2x^2 - x - 1}{x^2 - x + 1}$

$y =$ $x +$ [2]

11. Gestel

Suppose

$\int_1^2 f(x) dx = 5, \quad \int_1^4 f(x) dx = 8, \quad \int_3^4 f(x) dx = 6,$

Dan is / Then

$\int_2^3 f(x) dx =$ [2]

12. Bepaal die volgende integrale

Determine the following integrals:

(a) $\int (6x^2 - 5x + 1) dx =$ [2]

(b) $\int_0^3 \sqrt{9 - x^2} dx =$ [2]

(c) $\int_0^{\pi/6} \tan^2 \theta d\theta =$ [2]

(d) $\int_0^2 |x(x-1)| dx =$ [2]

13*. Hoeveel nulle is daar an die einde van die desimaaluitdrukking van $100!$?

How many zeroes are there at the end of the decimal expansion of $100!$?

[2]

Voorletters en van : Studentenr. :
Initials and surname : *Studentno.* :

Wiskunde / *Mathematics* 114

Eerste Eksamen / *First Exam*

Tweede Vraestel : Lang Vrae / *Second Paper: Long Questions*

10 Junie / *June* 2013

Tyd / *Time*: 2h

Volpunte / *Full marks*: 60

Beskikbare punte: / *Available marks*: 62

INSTRUKSIES:

INSTRUCTIONS:

- | | |
|--|--|
| • Hierdie vraestel bestaan uit 12 genommere bladsye. | <i>This paper consists of 12 numbered pages.</i> |
| • Skryf jou antwoorde direk op die vraestel in blou of swart ink. | <i>Write your answers directly on the question paper in blue or black ink.</i> |
| • Gebruik die agterkant van die bladsy vir jou rofwerk. As jy ekstra papier vir rofwerk benodig, mag jy een eksamenantwoordboekie gebruik, maar dit sal nie nagesien word nie. | <i>Use the back of the page for your rough work. If you need extra paper for rough work, you may use one exam answer booklet, but it will not be marked.</i> |
| • Geen boek of geskrif van enige aard mag in die toetslokaal ingebring word nie. | <i>No book or any written material may be brought into the examination room.</i> |
| • Geen deel van hierdie antwoordstel mag uitgeskeur word nie. | <i>No part of this answer sheet may be removed from the examination room.</i> |
| • Geen sakrekenaars toegelaat nie. | <i>No calculators allowed.</i> |
| • Vrae wat met 'n * gemerk is, is meer uitdagend. | <i>Questions marked with a * are more challenging.</i> |

1. Bereken die volgende afgeleide:

Compute the following derivative:

$$\frac{d}{dt} \frac{t \sin t}{t^2 + 1}$$

[4]

2. Skets die volgende versameling in die komplekse vlak:

Sketch the following set in the complex plane:

$$\{z \in \mathbb{C} \mid \operatorname{Im}(z^2) > 0, |z| \leq 1\}$$

[3]

3. Los op vir x :

Solve for x :

$$|x - 3| < 2x - 4$$

[4]

4. Beskou die volgende vergelyking:

Consider the following equation:

$$x^3 + x + 1 = 0.$$

- (a) Gebruik die Tussenwaardestelling om te wys 'n oplossing op die interval $[-1, 0]$ bestaan.

Use the Intermediate Value Theorem to show that a solution exists on the interval $[-1, 0]$. [2]

- (b) f is differensiëerbaar op \mathbb{R} .

f is differentiable on \mathbb{R} .

[3]

- (c) As ons Newton se metode, met beginwaarde $x_1 = 0$, gebruik om die oplossing te benader, wat is x_3 ?

If we use Newton's method, with initial value $x_1 = 0$, to approximate this solution, what is x_3 ? [4]

5. Verduidelik hoe om die grafiek van $y = x^2$ te verskuif om die grafiek van $y = x^2 + 4x$ te verkry.

Explain how to shift the graph of $y = x^2$ to obtain the graph of $y = x^2 + 4x$.

[3]

6. Beskou die volgende funksie:

Consider the following function:

$$f(x) = \begin{cases} \sin x & \text{as / if } x < 0 \\ mx + c & \text{as / if } x \geq 0 \end{cases}$$

Bepaal alle waardes van $m, c \in \mathbb{R}$ waarvoor

Determine all values of $m, c \in \mathbb{R}$ for which

- (a) f is kontinu op \mathbb{R} ;

f is continuous on \mathbb{R} ;

[2]

- (b) f is differensiëerbaar op \mathbb{R} .

f is differentiable on \mathbb{R} .

[3]

7. Beskou die funksie f en sy afgeleides:

Consider the function f and its derivatives:

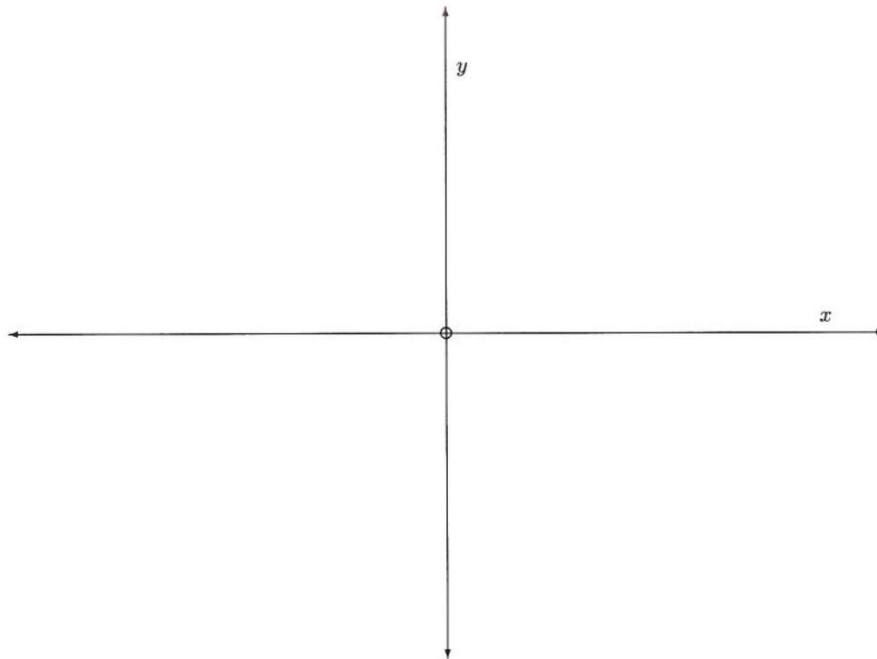
$$f(x) = \frac{(x+1)^2}{x-3}, \quad f'(x) = \frac{x^2 - 6x - 7}{(x-3)^2}, \quad f''(x) = \frac{32}{(x-3)^3}.$$

Voltooi die volgende tabel met inligting oor f , en skets 'n grafiek van $y = f(x)$ op die gegewe koördinaatstelsel wat al hierdie inligting in ag neem. Alle bewerkings moet op die volgende bladsy getoon word.

Complete the following table with information on f , and sketch a graph of $y = f(x)$ on the given coordinate system which takes all this information into account. All computations must be shown on the following page.

[16]

Definisieversameling / Domain	
x -afsnit(te) / x -intercept(s)	
y -afsnit / y -intercept	
Vertikale asimptote / Vertical asymptotes	
Horisontale asimptote / Horizontal asymptotes	
Stygend op / Increasing on	
Dalend op / Decreasing on	
Lokale maksima / Local maxima	
Lokale minima / Local minima	
Konkaaf opwaarts (na bo) op / Concave up on	
Konkaaf afwaarts op / Concave down on	
Buigpunte (infleksiepunte) / Inflection points	



Berekening vir Vraag 3: / Calculations for Question 3:

$$f(x) = \frac{(x+1)^2}{x-3}, \quad f'(x) = \frac{x^2 - 6x - 7}{(x-3)^2}, \quad f''(x) = \frac{32}{(x-3)^3}.$$

8. Die bestuurder van 'n kompleks van 75 woonstelsse weet dat alle woonstelsse verhuur sal wees by 'n huurprys van R5000. 'n Markopname het getoon dat vir elke R200 verhoging van die huurpreis een addisionele woonstel leeg sal staan. By watter huurpreis sal die totale inkomste die grootste wees? *The manager of a 75-unit block of flats knows that all flats will be occupied if the rent is set at R5000. A market survey has shown that one additional unit will remain vacant for each R200 increase in rent. What rent should the manager set to maximise revenue?* [5]

9. Skryf neer 'n Riemann som wat die volgende integraal voorstel. Moenie die integraal bereken nie. *Write down a Riemann sum which represents the following integral. Do not evaluate the integral.* [3]

$$\int_{\pi}^{2\pi} \frac{\sin x}{x} dx$$

10. Gebruik die knypstangstelling om te bewys dat *Use the Squeeze Theorem to prove that* [2]

$$\lim_{x \rightarrow \pi} (x - \pi)^2 \cdot \cos\left(\frac{1}{x - \pi}\right) = 0$$

11. Bewys dat $n! > 2^n$ vir alle positiewe heelgetalle $n \geq 4$. *Prove that $n! > 2^n$ for all positive integers $n \geq 4$.* [5]

- 12*. Bewys die volgende: *Prove the following:*

$$\int_0^{\frac{\pi}{2}} x \sin^2 x dx \leq \frac{\pi^2}{8}$$

(Wenk: moenie probeer die integraal op te los nie.) *(Hint: Do not try to solve the integral.)*

[2]

- 13*. **Definisie.** Gestel $f : [a, \infty) \rightarrow \mathbb{R}$ is 'n funksie en $L \in \mathbb{R}$. Dan skryf ons **Definition.** Suppose $f : [a, \infty) \rightarrow \mathbb{R}$ is a function and $L \in \mathbb{R}$. Then we write

$$\lim_{x \rightarrow \infty} f(x) = L$$

$$\lim_{x \rightarrow \infty} f(x) = L$$

as vir elke $\epsilon > 0$ 'n $N \in \mathbb{R}$ bestaan sodat, $|f(x) - L| < \epsilon$ vir elke $x > N$.

if for every $\epsilon > 0$ there exists $N \in \mathbb{R}$ such that $|f(x) - L| < \epsilon$ for every $x > N$.

Bewys vanuit die bostaande definisie dat

Prove from the above definition that

$$\lim_{x \rightarrow \infty} \frac{1}{x^2} = 0.$$

[2]

